# Math 220/903&904-Homework 4

**Due Wednesday 10/07 at the beginning of class**

## PART A

Problems from the textbook:

**Section 2.2** # 16(a,b); 23

**Section 2.3** # 2, 4, 5(b,c,e,f), 11, 14, 23.

## PART B

1. Determine the truth or falsehood of the following statements. (Write TRUE or FALSE for each statement.)

   (a) $A \times B = B \times A$ for all nonempty sets $A$ and $B$.

   (b) If $A$ is not a subset of $B$ and $B$ is not a subset of $A$, then $A \cap B = \emptyset$.

   (c) For all sets $A$, $B$, and $C$, $A \cup (B \cap C) = (A \cap C) \cup (B \cap C)$.

   (d) $7 \notin \{\{-1, 7\}, \{-7, 2015, 0\}, \{1, 2\}\}$.

   (e) $A \cup A = A \cap A$ for all sets $A$.

   (f) If $A = \{a, \{a, b, c\}\}$ and $B = \{\{c, d\}, \{a, b, c, d\}\}$ then $|A| = |B|$. .

   (g) If $\{1\} \in P(A)$, then $1 \in A$ and $\{1\} \notin A$.

2. For the sets $A = \{a, b\}$ and $B = \{0, 1\}$ form the following Cartesian products:

   (a) $B \times A$

   (b) $B \times A \times B$.

3. Let $A = \{a, b, c\}$.

   (a) Write out all the different partitions of the set $A$.

   (b) Write out the power set, $P(A)$, for $A$.

4. Give an example of two different partitions of the set $\{x | x \text{ is an integer}\}$.

5. For each $n \in \mathbb{Z}^+$, define $A_n = \{n, 2n\}$. Let $I = \{1, 2, 4\}$. Find $\bigcup_{\alpha \in I} A_\alpha$.

6. For each $n \in \mathbb{Z}^+$, define $A_n = \left\{ x \in \mathbb{R} | \ -\frac{1}{n} \leq x \leq 2 - \frac{1}{n} \right\}$. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i \in \mathbb{Z}^+} A_i$.

7. Let A, B, and C be nonempty sets. Prove the following statements.

   (a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

   (b) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.