

Math 220 – Homework 4

Due Thursday 2/14 at the beginning of class

Total points: 110 (Writing portion 110 pts) (Problems marked by * will count toward writing portion.)

PART A

Problems from the textbook:

• Section 2.2	problem	3*	5*
	points	10	10

1. *[10 points] Let $x \in \mathbb{Z}$. Prove that if $7x + 5$ is odd, then x is even.
2. *[10 points] Let $x, y \in \mathbb{Z}$. Prove that if $3x + 7y$ is even, then x and y are of the same parity.
3. *[10 points] Let $a, b, c \in \mathbb{Z}$. Prove that if $a \nmid bc$, then $a \nmid b$ and $a \nmid c$.
4. * [10 points] Prove that if a and b are odd integers, then $4 \nmid (a^2 + b^2)$.
5. * [10 points] Prove that if x is an integer, then x^3 has the same parity as x .
6. (a) * [10 points] Let $n \in \mathbb{Z}$. Prove that if $2 \mid (n^2 - 5)$, then $4 \mid (n^2 - 5)$.
 (b) [5 points] Give an example of an integer n such that $2 \mid (n^2 - 5)$, but $8 \nmid (n^2 - 5)$
7. Consider the statement:

“If the product of two integers is even, then at least one of these integers is even.”

 (a) [5 points] Rewrite the statement in symbols.
 (b) * [10 points] Give a formal proof.
8. * [10 points] Let a be a positive real number. Prove that there is a unique positive real number x such that $x^4 - a^4 = 0$.