Math 220/970(HNR) – Homework 5

Due Thursday 10/06 at the beginning of class

PART A

Problems from the textbook:

Section 5.2 # 3; 4(b); 2(b).

PART B

1. Prove that the equation $x^5 + 2x - 5 = 0$ has a *unique* real number solution between $x = 1$ and $x = 2$.
2. Prove that the equation $\cos^{2016}(x) - 4x + \pi = 0$ has a real number solution between $x = 0$ and $x = 4$. (Note: do not use a calculator! You may assume that $\cos^{2016}(x)$ is continuous on $[0, 4]$.)
3. Let $a, b, c \in \mathbf{Z}$. Disprove the following statements.
   (a) If $a|c$ and $b|c$, then $ab|c$.
   (b) If $a|b$ and $b|a$ then $a = b$.
4. Prove the following statement: *"No odd integer can be expressed as the sum of three integers."*
5. Suppose $n \in \mathbf{Z}$. Prove that $15|n$ if and only if $5|n$ and $3|n$.
6. Assume that $x, y \in \mathbf{Z}$. Prove that if $x + y$ is odd, then $x^2 + y^2$ is odd.
7. Prove by induction that for every positive integer n the following statements hold:
   (a) $2 + 6 + 10 + \ldots + (4n - 2) = 2n^2$.
   (b) $n^3 + 2n$ is divisible by 3. (Hint: $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$)
   (c) $\dfrac{1}{2 \cdot 3} + \dfrac{1}{3 \cdot 4} + \ldots + \dfrac{1}{(n+1)(n+2)} = \dfrac{n}{2(n+2)}$.
   (d) $26|(3^{3n} - 1)$.
   (e) 13 is a factor of $17^n - 4^n$.