Math 220(HNR) – Homework 5

Due Thursday 02/23 at the beginning of class **Total points = 210**

PART A

Problems from the textbook:

Section 1.4 # 17 [10 points], 21 [10 points]

Section 5.2 # 3 [10 points]; 2(b) [10 points]; 4(b) [10 points].

PART B

1. [12 points] Prove that the equation $x^5 + 2x - 5 = 0$ has a *unique* real number solution between $x = 1$ and $x = 2$.
2. [10 points] Prove that the equation $\sin^{2016}(x) - 4x + \pi = 0$ has a real number solution between $x = 0$ and $x = 4$. (You may assume that $\sin^{2016}(x)$ is continuous on $[0, 4]$.)
3. [12 points] Let $a, b, c \in \mathbf{Z}$. Determine the truth or falsehood of the following statements. If the statement is true, prove it; otherwise, provide a counterexample.
   (a) If $a|c$ and $b|c$, then $ab|c$.
   (b) If $a|b$ and $b|a$ then $a = b$.
4. [10 points] Prove the following statement: *"No odd integer can be expressed as the sum of three even integers."*
5. [14 points] Suppose $n \in \mathbf{Z}$. Prove that $15|n$ if and only if $5|n$ and $3|n$.
6. [10 points] Assume that $x, y \in \mathbf{Z}$. Prove that if $x + y$ is odd, then $x^2 + y^2$ is odd.
7. Prove by induction that for every positive integer n the following statements hold:
   (a) [10 points] $n^3 + 2n$ is divisible by 3. (Hint: $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$)
   (b) [10 points] $\dfrac{1}{2\cdot 3} + \dfrac{1}{3\cdot 4} + \ldots + \dfrac{1}{(n+1)(n+2)} = \dfrac{n}{2(n+2)}$.
   (c) [10 points] $7|(2^{3n} - 1)$.
   (d) [10 points] 3 is a factor of $7^n - 4^n$.
8. [10 points] Disprove the following statement:
   `There exist odd integers` a `and` b `such that` $4|(7a^2 - b^2)$.
9. Prove the statement ``If n is an odd integer, then $27n + 5$ is even.'' by
   (a) [6 points] a direct proof;
   (b) [7 points] a proof by contrapositive;
   (c) [7 points] a proof by contradiction.
10. [10 points] Use proof by contradiction to prove that if a and b are odd integers, then $4 \nmid (a^2 + b^2)$.
11. [12 points] Let $x, y \in \mathbf{R}$. Proof that if $xy \neq 0$, then $x \neq 0$ by using more than one method of proof.