

Math 220 – Homework 6

Due Thursday 2/28 at the beginning of class

Total points: 182

(Writing portion 45 pts)

PART A

Problems from the textbook:

• Section 4.1	problem	1(a,b,d,g,j)	4*	5*	6*
	points	20	10	10	10

PART B

1. [10 points] Which of the following sets are equal? Justify your answers.

$$A = \{n \in \mathbb{Z} \mid |n| < 2\}, \quad B = \{n \in \mathbb{Z} \mid n^3 = n\}, \quad C = \{n \in \mathbb{Z} \mid n^2 \leq n\},$$

$$D = \{n \in \mathbb{Z} \mid n^2 \leq n\}, \quad E = \{-1, 0, 1\}, \quad F = \{t \in \mathbb{Z} \mid |t| < 2\}$$

2. [18 points] Let $U = \{a, 3a, 5a, \dots, 15a\}$ be the universal set and let $A = \{a, 5a, 9a, 13a\}$, and $B = \{3a, 9a, 15a\}$. Determine the following:

(a) $\bar{A} \cup B$ (b) $A \cap B$ (c) $A - B$ (d) $B - A$ (e) $\bar{A} \cap \bar{B}$.

3. [10 points] Let A , B and C be nonempty subsets of a universal set U . Draw a Venn diagram for each of the following set operations.

(a) $B \cup (A - C)$ (b) $A \cap (C - B)$

4. * [15 points] Let A , B , and C be nonempty subsets of a universal set U . Disprove the following statements:

(a) If $A \cap B = A \cap C$, then $B = C$.

(b) If $A - B = C - B$, then implies $A = C$.

(c) If A is not a subset of B and B is not a subset of A , then $A \cap B = \emptyset$.

5. [9 points] Describe the following sets by listing their elements.

(a) The set of all remainders when a positive integer is divided by 7.

(b) The set of all integers of absolute value less or equal than 2.

(c) $A = \{x \in \mathbb{R} \mid x^3 - x = 0\}$

6. [12 points] Describe the following sets by listing enough elements to indicate a pattern for all elements of the set.

(a) The set of all remainders when a natural number is divided by 2018.

(b) The set of all numbers x for which $\tan x$ is undefined.

(c) $A = \{3q + 1 \mid q \in \mathbb{Z}\}$

7. [6 points] Describe the following sets using a set-builder notation. Namely, write them in the form $\{x \in D \mid \dots\}$ for the appropriate set D .

(a) The set of all rational numbers between 0 and 1 inclusive.

- (b) The set of all numbers x for which $\tan x = 0$.
8. [12 points] Let $U = \mathbb{R}$ be the universal set. Consider $A = \{x \in \mathbb{R} \mid |2x + 3| \geq 19\}$ and $B = \{x \in \mathbb{R} \mid |x| \leq 3\}$.
- (a) Express the sets A and B using interval notation (as an interval or a union of intervals).
- (b) Determine $\overline{A} \cap \overline{B}$ as an interval or a union of intervals.
9. [10 points] Given $A = \{x \in \mathbb{Z} \mid |x| > 10\}$. Compute the complement of A , if (a) $U = \mathbb{Z}$ (b) $U = \mathbb{R}$.
10. [10 points] Given $A = \{x \in \mathbb{R} \mid |x| > 10\}$ and $B = \{x \in \mathbb{R} \mid 0 < |x| \leq 12\}$. Compute $A - B$ and $B - A$.
11. [20 points] Assume that A and B are sets and P and Q are propositions. Characterize the following expressions as either
- (i) a proposition/statement
(ii) not a proposition/statement, but an expression that makes sense mathematically.
(iii) an expression that makes no sense mathematically.

(Write i, ii, or iii as an answer for each item.)

- (a) i / ii / iii $(A \cap B) \subset \overline{A}$
- (b) i / ii / iii $B \Rightarrow A$
- (c) i / ii / iii $(A \subset B) \Rightarrow Q$
- (d) i / ii / iii $P \subseteq Q$
- (e) i / ii / iii $P = Q$
- (f) i / ii / iii $B \cap \overline{A} = \overline{B}$
- (g) i / ii / iii $P \cap Q$
- (h) i / ii / iii $A + B$
- (i) i / ii / iii $(\exists x \in P)[x \in A \cup B]$
- (j) i / ii / iii $A \vee B$