Math 220 − Homework 7

Due Thursday 03/23 at the beginning of class

Total points=197

PART A

Problems from the textbook:

Section 2.1 # 7(e, f, h) 15pts ; 8(b,c, d) 15pts ; 19(b, c) 10pts ; 20(c, f) 10pts .

Section 2.2 # 4(b, e) 6pts ; 5(b, e) 6pts ; 6 16pts , 26 22pts

Section 2.3 # 2 8pts , 4(a,b,c) 14pts , 5(b,c,e,f) 8pts .

PART B

1. For the sets $A = \{x \in \mathbb{Z}| \ 2 \leq x < 4\}$ and $B = \{x \in \mathbb{R}|x^4 = 1\}$ form the following Cartesian products:

   (a) 4pts $B \times A$

   (b) 8pts $B \times A \times B$.

2. 14pts Let A, B, and C be nonempty sets. *Determine the truth or falsehood of the following statements.* (Write TRUE or FALSE for each statement.)

   (a) $A − A = \emptyset$.

   (b) $A \subset A$.

   (c) $A \cup (B \cap C) = (A \cap C) \cup (B \cap C)$.

   (d) $A \cup A = A \cap A$ for all sets A.

   (e) If $|A| = |B|$ then $A \times B = B \times A$.

   (f) $A \times B = B \times A$ for all nonempty sets A and B.

   (g) If $\{1\} \in P(A)$, then $1 \in A$ and $\{1\} \notin A$.

3. 10pts Let $A = \{x \in \mathbb{N}| \ 1 \leq x < 5\}$ and $P(A)$ be a power set of A. Determine the truth or falsehood of the following statements. (Write TRUE or FALSE for each statement.)

   (a) $A \subset P(A)$.

   (b) $\{2\} \in P(A)$.

   (c) $[3, 4] \subseteq A$.

   (d) $|P(A)| = 32$

   (e) $\emptyset \subseteq P(A)$ and $\emptyset \in P(A)$.

4. 8pts Let $U = \mathbb{R}$ be the universal set. Consider $A = \{x \in \mathbb{R}| \ |2x + 3| \geq 19\}$ and $B = \{x \in \mathbb{R}| \ |x| \leq 3\}$.

   (a) Express the sets A and B using interval notation (as an interval or a union of intervals).

   (b) Determine $\overline{A} \cap \overline{B}$ as an interval or a union of intervals.

5. 15pts Let $U = \{x, y, 1, 2, 3\}$ be the universal set and let $A = \{x, y, 1, 2\}$, $B = \{2, 3\}$, $C = \{1, 3, x, y\}$. Determine the following (show all intermediate steps):

   (a) $\overline{A} \cup (B \cap C)$

   (b) $\overline{B \cup C} \cap U$

   (c) $\overline{(A \cup B) − (B \cap C)}$

6. 8pts Find $|A|$, where $A = \{(x, y) \in \mathbf{Z} \times \mathbf{Z}| \ |x| + |y| = 3\}$.