

Math 220/903&904-Homework 7

Due Wednesday 10/28 at the beginning of class

PART A

Problems from the textbook:

Section 3.3 # 1(b,c); 2(b), 10(c,d), 12, 14, 16, 20

PART B

1. Let $A = \mathbb{R} - \{1\}$ and define $f \in F(A)$ by $f(x) = \frac{x}{x-1}$.
 - (a) Prove by definition that f is bijective.
 - (b) Determine $f^{-1}(x)$.
 - (c) Determine the composition $f \circ f \circ f$.
2. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$, and $C = \{w, x, y, z\}$. Consider the functions $f \in F(A, B)$ and $g \in F(B, C)$, where

$$f = \{(1, b), (2, c), (3, c), (4, a)\}, \quad g = \{(a, x), (b, y), (c, x)\}.$$

(i.e. $f(1) = b, f(2) = c$, etc.)

- (a) Find $f^{-1}(\{a, c\})$.
 - (b) Determine $g \circ f$.
3. The functions $f, g \in F(\mathbf{R})$ defined by $f(x) = 2x + 1$ and $g(x) = 3x - 5$ are bijective. Determine the inverse function of $g \circ f^{-1}$.
 4. Let $f \in F(A, B)$ and $X \subseteq A$.
 - (a) Prove that $X \subseteq f^{-1}(f(X))$.
 - (b) Give an example of a function $f \in F(A, B)$ for some A, B and a subset X of A such that $X \neq f^{-1}(f(X))$.
 5. Let $a, b \in \mathbb{R} - \{0\}$ and let functions $f, g \in F(\mathbf{R})$ be defined by

$$f(x) = ax + b, \quad g(x) = x + \frac{b}{a}.$$

Compute the *inverse* function of $g \circ f^{-1}$.