Math 220 (HNR) – Homework 8

Due Thursday 11/03 at the beginning of class

PART A

Problems from the textbook:

Section 3.1 # 1, 2, 7, 10(a)

Section 3.2 # 1(e); 2(e); 9; 10, 12(b,c); 13(c,e); 14(a,e);

PART B

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2016 - 4x$. Prove that $Imf = \mathbb{R}$.
2. Let $f \in F(\mathbb{R})$ be defined by $f(x) = -x^{2n}$, where $n \in \mathbb{Z}^+$, and $S = \{y \in \mathbb{R} |\ y \le 0\}$. Prove that $Imf = S$.
3. Determine whether the following function is injection. Give a formal proof of your answer.
   (a) $f \in F(\mathbb{R})$ defined by $f(x) = 16x^{16} - 14x^{14} - 2x^2 + 1$
   (b) $f \in F(\mathbb{Z})$ defined by $f(n) = \begin{cases} n + 2016, & \text{if} \quad n \in \mathbb{E} \\ -n + 2016, & \text{if} \quad n \in \mathbb{O} \end{cases}$
4. Determine whether the function $f \in F(\mathbb{Z})$ defined by $f(n) = \begin{cases} 2n, & \text{if} \quad n \in \mathbb{E} \\ -n + 22, & \text{if} \quad n \in \mathbb{O} \end{cases}$ is surjective.

   Give a formal proof of your answer.
5. Let $f : \mathbb{Z} \to \mathbb{R}$ and let $g : \mathbb{Z} \to \mathbb{R}$ be defined by $f(n) = \cos(\pi n)$ and $g(n) = (-1)^n$.
   (a) Find $Im(f)$ and $Im(g)$ and represent your answers using roster notation.
   (b) Find graphs G_f and G_g and show that $G_f = G_g$.