## Math 220 - Homework 8

Due Thursday 03/30 at the beginning of class
Total points $=155$

## PART A

Problems from the textbook:
Section $2.2 \# 15(\mathrm{a}) 10 \mathrm{pts}, 17(\mathrm{a}) 10 \mathrm{pts}, 2310 \mathrm{pts}, 25(\mathrm{~b}) 10 \mathrm{pts}$
Section 3.1 \# 1 10pts, $210 \mathrm{pts}, 3(\mathrm{a}, \mathrm{b}, \mathrm{d}, \mathrm{f}) 420 \mathrm{pts}, 75 \mathrm{pts}$

## PART B

1. Let A, B, and C be nonempty sets. Prove the following statements.
(a) 10pts $A \times(B \cap C)=(A \times B) \cap(A \times C)$.
(b) $10 \mathrm{pts}(A \times B) \cap(C \times D)=(A \cap C) \times(B \cap D)$.
(c) 10 pts If $A \subseteq B$, then $A \cup C \subseteq B \cup C$.
(d) $10 \mathrm{pts} A \times(B-C)=(A \times B)-(A \times C) .($ Hint: $(x \in A) \wedge(y \notin B) \Rightarrow((x, y) \notin A \times B)$.
2. 10pts Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x)=2016-4 x$. Prove that $\operatorname{Im} f=\mathbb{R}$.
3. 10pts Let $f \in F(\mathbb{R})$ be defined by $f(x)=-x^{2 n}$, where $n \in \mathbb{Z}^{+}$, and $S=\{y \in \mathbb{R} \mid y \leq 0\}$. Prove that $\operatorname{Im} f=S$.
4. Let $f: \mathbb{Z} \rightarrow \mathbb{R}$ and let $g: \mathbb{Z} \rightarrow \mathbb{R}$ be defined by $f(n)=\cos (\pi n)$ and $g(n)=(-1)^{n}$.
(a) 5pts Find $\operatorname{Im}(f)$ and $\operatorname{Im}(g)$ and represent your answers using roster notation. (No formal proofs are necessary).
(b) 5 pts Find graphs $G_{f}$ and $G_{g}$ and show that $G_{f}=G_{g}$.
