## Math 220 – Homework 8

## Due Thursday 03/30 at the beginning of class

Total points=155

## PART A

Problems from the textbook:

Section	2.2	# 15	$\mathbf{b}(\mathbf{a})$ 10	pts	, 17(a)	) $10 \text{pts}, 23$	B 10pts	s], 2	25(b)	10pts
Section	3.1	#1	10pts	, 2	10pts	, 3(a,b,d,f)	20pts	, 7	5pts	]

## PART B

1. Let A, B, and C be nonempty sets. Prove the following statements.

(a) 
$$|10\text{pts}| A \times (B \cap C) = (A \times B) \cap (A \times C)$$

- (b) 10pts  $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D).$
- (c) 10pts If  $A \subseteq B$ , then  $A \cup C \subseteq B \cup C$ .
- (d) 10pts  $A \times (B C) = (A \times B) (A \times C)$ . (Hint:  $(x \in A) \land (y \notin B) \Rightarrow ((x, y) \notin A \times B.)$ )
- 2. |10pts | Let  $f : \mathbb{R} \to \mathbb{R}$  be defined by f(x) = 2016 4x. Prove that  $Imf = \mathbb{R}$ .
- 3. 10pts Let  $f \in F(\mathbb{R})$  be defined by  $f(x) = -x^{2n}$ , where  $n \in \mathbb{Z}^+$ , and  $S = \{y \in \mathbb{R} \mid y \leq 0\}$ . Prove that Imf = S.
- 4. Let  $f: \mathbb{Z} \to \mathbb{R}$  and let  $g: \mathbb{Z} \to \mathbb{R}$  be defined by  $f(n) = \cos(\pi n)$  and  $g(n) = (-1)^n$ .
  - (a) 5pts Find Im(f) and Im(g) and represent your answers using roster notation. (No formal proofs are necessary).
  - (b) 5pts Find graphs  $G_f$  and  $G_g$  and show that  $G_f = G_g$ .