Math 220 – Homework 9 HNR

Due Thursday 11/08 at the beginning of class

Total points: 224

PART A

Problems from the textbook:
p.236 # 9.38 [8 points]

PART B

1. [10 points] For a real number r, define S_r to be the interval $[r-1, r+2]$. Let $A = \{1,3,4\}$. Write the sets $\bigcup_{\alpha\in A} S_\alpha$ and $\bigcap_{\alpha\in A} S_\alpha$ in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

2. [10 points] Let $K = \{a,b,c\}$, $L = \{b,d,e\}$, $M = \{b,e,f\}$ and $S = \{K, L, M\}$. Write the sets $\bigcup_{X\in S} X$ and $\bigcap_{X\in S} X$ in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

3. [30 points] Let $i \in \mathbb{Z}$ and $A_i = \{i-1, i+1\}$. Write the following sets in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

   (a) $\bigcup_{i=1}^{5} A_{2i}$ **(b)** $\bigcup_{i=1}^{250} A_{2i}$ **(c)** $\bigcup_{i=1}^{5} (A_i \cap A_{i+1})$ **(d)** $\bigcup_{i=1}^{250} (A_i \cap A_{i+1})$ **(e)** $\bigcup_{i=1}^{5} (A_{2i-1} \cap A_{2i+1})$
   (f) $\bigcup_{i=1}^{250} (A_{2i-1} \cap A_{2i+1})$

4. [15 points] Repeat the previous problem for $A_i = [i-1, i+1]$.

5. [15 points] Given $I = \{1,2,3,\dots,2018\}$. For each $i \in I$ define $B_i = \{i, i+1\}$. Write the following in a simpler form (as either an interval or a finite set of points). Show all steps leading to your final answer.

   (a) $\bigcap_{i\in I} B_i$ **(b)** $\bigcap_{i=j}^{j+1} B_i$ **(c)** $\bigcup_{i=j}^{k} B_i$, where $1 \le j < k \le 2018$

6. Let $A = \{x,y,z,u,v\}$, $B = \{a,b,c,d\}$, and $C = \{5,6,7,8,9\}$.
   (a) [9 points] Write out three functions with domain A and codomain B making at least one of the functions have the property that its range coincides with its codomain (represent all functions by their graphs (see Definition 3 in notes)).
   (b) [9 points] Write out two functions with domain B and codomain C(represent all functions by their graphs (see Definition 3 in notes)). Explain why you cannot define a function between these two sets for which the range equals its codomain.

7. * [10 points] Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2018 - 4x$. Prove that ran$f = \mathbb{R}$.

8. * [10 points] Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^4$ and $S = \{y \in \mathbb{R} \mid y \ge 0\}$. Prove that ran$f = S$.

9. Let $X = \{x \in \mathbb{R} \mid x \ne -5\}$ and $f : X \to \mathbb{R}$ be defined by $f(x) = \dfrac{3x-1}{x+5}$.

(a) [5 points] Determine the range of f.

(b) * [10 points] Prove that your answer for ran$f$ is correct.

10. Express each of the following functions as a composition $f = g \circ h$. Be sure to give appropriate sets A, B, and C such that $h : A \to B$ and $g : B \to C$. Note that neither g nor h should be an identity functions, but there may be many possible answers.

    (a) [7 points] $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sqrt[3]{e^{x^3} + 8}$

    (b) [7 points] $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \ln(x^2 + 1)$

    (c) [7 points] $f : \mathbb{Z} \to \mathbb{R}$ defined by $f(x) = \sin(\pi x + 1)$

11. * [10 points] Determine whether the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = \begin{cases} 2n, & \text{if } n \in \mathbb{E} \\ -n + 22, & \text{if } n \in \mathbb{O} \end{cases}$ is surjective. Give a formal proof of your answer.

12. [32 points] Find $\bigcup\limits_{i \in I} A_i$ and $\bigcap\limits_{i \in I} A_i$ if

    (a) $I = \mathbb{Z}^+$ and $A_i = (-i, i)$.

    (b) $I = \{2, 3, 4, 5, ...\}$ and $A_i = [\frac{1}{i}, i)$.

    (c) $I = \mathbb{Z}^+$ and $A_i = [0, 1 - \frac{1}{i}]$.

    (d) $I = \mathbb{Z}^+$ and $A_i = [1 - \frac{1}{i}, 3 - \frac{1}{i})$.

13. [10 points] Find $f \circ g$ if $f, g : \mathbb{Z} \to \mathbb{Z}$ by $f(n) = 2n + 3$ and $g(n) = \begin{cases} 2n - 1, & \text{if } n \in \mathbb{E} \\ n + 1, & \text{if } n \in \mathbb{O} \end{cases}$

14. [10 points] Let $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ x - 2 & \text{if } x < 0 \end{cases}$ and $g : \mathbb{R} \to \mathbb{R}$ by $g(x) = \begin{cases} x + 3 & \text{if } x \geq 4 \\ 2x & \text{if } x < 4 \end{cases}$

    Find $f \circ g$.