Math 220-Homework 9

Due Thursday 04/09 at the beginning of class

PART A

Problems from the textbook:

- Section 5.3 # 1b, 3, 11, 12, 13, 15

PART B

1. Let $a, b, c \in \mathbf{Z}$. Determine the truth or falsehood of the following statements.
   (a) $\gcd(a, 0) = a$.
   (b) Let a and b be not both zero. Then $\gcd(a, b) = \gcd(-a, b)$.
   (c) The set $\mathbf{Z} - \mathbf{Z}^+$ of integers is closed with respect to multiplication.
   (d) $0|b$ only if $b = 0$.
   (e) If $a|c$ and $b|c$, then $ab|c$.
   (f) If $a|b$ and $b|a$ then $a = b$.
2. Prove by induction that for every positive integer n the following statements hold:
   (a) $2 + 6 + 10 + \ldots + (4n - 2) = 2n^2$.
   (b) $n^3 + 2n$ is divisible by 3. (Hint: $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$)
   (c) $\dfrac{1}{2 \cdot 3} + \dfrac{1}{3 \cdot 4} + \ldots + \dfrac{1}{(n + 1)(n + 2)} = \dfrac{n}{2(n + 2)}$.
   (d) $3|(2^{2n} - 1)$.
   (e) 5 is a factor of $7^n - 2^n$.
3. (a) Use the Euclidean Algorithm to determine $\gcd(374, 946)$.
   (b) Find integers x and y such that $374x + 946y = \gcd(374, 946)$.
4. Prove that if a, b, and c are integers such that $a|b$ and $a|c$ then $a|(2014b - 2015c)$.
5. Find integers x and y such that $51x + 288y = \gcd(51, 288)$.