## Math 220 – Homework 9

## Due Thursday 3/28 at the beginning of class

Total points: 148

(Writing portion 115 pts)

## PART A

Problems from the textbook:

- Section 5.2  $\begin{array}{c|c|c|c|c|c|c|c|c|} problem & 1(a) & 1(b) & 2\\ \hline points & 8 & 10 & 10 \\ \end{array}$
- Section 5.3 # 3(a,c)\* [40 points]

## PART B

- 1. \* [10 points] Let  $f, g : \mathbb{R} \to \mathbb{R}$  are defined by  $f(x) = 2x^2 1$  and g(x) = 3x + 5. Determine  $(g \circ f)(1)$  and  $(f \circ g)(1)$ .
- 2. \* [10 points] Let  $f : \mathbb{R} \to \mathbb{R}$  be defined by f(x) = 3x 2019. Prove that ran $f = \mathbb{R}$ .
- 3. \* [10 points] Let  $f : \mathbb{R} \to \mathbb{R}$  be defined by  $f(x) = 6x^6$  and  $S = \{y \in \mathbb{R} | y \ge 0\}$ . Prove that ranf = S.
- 4. \* [5 points] Let  $f: [-1,\infty] \to \mathbb{R}$  be defined by  $f(x) = \sqrt[4]{1+x}$  and  $S = [0,\infty)$ . Prove that  $S \subseteq \operatorname{ran} f$ .

5. Let  $X = \{x \in \mathbb{R} | x \neq -5\}$  and  $f : X \to \mathbb{R}$  be defined by  $f(x) = \frac{3x-1}{x+5}$ .

- (a) [5 points] Determine the range of f.
- (b) \* [10 points] Prove that your answer for ran f is correct.
- 6. \* [20 points] A function  $f: \mathbb{Z} \to \mathbb{Z}$  is defined by f(n) = 7n + 3. Prove or disprove the following:
  - (a) f is injective.
  - (b) f is surjective.
- 7. \* [10 points] Determine whether the function  $f : \mathbb{Z} \to \mathbb{Z}$  defined by  $f(n) = \begin{cases} 2n, & \text{if } n \in \mathbb{E} \\ -n+22, & \text{if } n \in \mathbb{O} \end{cases}$  is surjective. Give a formal proof of your answer.