## Math 220 - Homework 9

## Due Thursday $3 / 28$ at the beginning of class

Total points: 148
(Writing portion 115 pts)

## PART A

Problems from the textbook:

- Section 5.2 | problem | $1(\mathrm{a})$ | $1(\mathrm{~b})$ | 2 |
| :---: | :---: | :---: | :---: |
|  | points | 8 | 10 |
- Section $5.3 \# 3(\mathrm{a}, \mathrm{c})^{*}[40$ points $]$


## PART B

1.     * $[10$ points $]$ Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x)=2 x^{2}-1$ and $g(x)=3 x+5$. Determine $(g \circ f)(1)$ and $(f \circ g)(1)$.
2.     * [10 points] Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x)=3 x-2019$. Prove that $\operatorname{ran} f=\mathbb{R}$.
3.     * $[10$ points $]$ Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x)=6 x^{6}$ and $S=\{y \in \mathbb{R} \mid y \geq 0\}$. Prove that ran $f=S$.
4.     * [5 points] Let $f:[-1, \infty] \rightarrow \mathbb{R}$ be defined by $f(x)=\sqrt[4]{1+x}$ and $S=[0, \infty)$. Prove that $S \subseteq \operatorname{ran} f$.
5. Let $X=\{x \in \mathbb{R} \mid x \neq-5\}$ and $f: X \rightarrow \mathbb{R}$ be defined by $f(x)=\frac{3 x-1}{x+5}$.
(a) [5 points] Determine the range of $f$.
(b) * [10 points] Prove that your answer for $\operatorname{ran} f$ is correct.
6.     * [20 points] A function $f: \mathbb{Z} \rightarrow \mathbb{Z}$ is defined by $f(n)=7 n+3$. Prove or disprove the following:
(a) $f$ is injective.
(b) $f$ is surjective.
7.     * [10 points $]$ Determine whether the function $f: \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(n)= \begin{cases}2 n, & \text { if } n \in \mathbb{E} \\ -n+22, & \text { if } n \in \mathbb{O}\end{cases}$ is surjective. Give a formal proof of your answer.
