Math 251 Sample EXAM III

LAST NAME (print)__________________________ FIRST NAME : _______________________

UIN: __________________________________SEAT#: _______

DIRECTIONS:

- The use of a calculator, laptop or computer is prohibited.
- In all problems present your solutions in the space provided.
- Be sure to read the instructions to each problem *carefully*.
- Use a pencil and be neat. If I can't read your answers, then I can't give you credit.
- *Show all your work* and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho\cos\phi$$

$$\mathrm{d}S = |\mathbf{n}(u,v)|\mathrm{d}A$$

$$\mathrm{d}\mathbf{S} = \hat{\mathbf{n}}\mathrm{d}S = \mathbf{n}(u,v)\mathrm{d}A$$

$$\oint_C \mathbf{F}\cdot \mathrm{d}\mathbf{r} = \iint_S \mathrm{curl}\mathbf{F}\cdot \mathrm{d}\mathbf{S}$$

Good Luck!

1. Convert the integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{-\sqrt{1-x^2-y^2}}^{0}\ln(4+x^2+y^2+z^2)\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$$

to an integral in spherical coordinates, but don't evaluate it.

WRITE YOUR ANSWER HERE:

2. Find the mass of a thin wire in the shape of C with the density $\rho(x,y,z)=7y^2z$ if C is given by $\mathbf{r}(t)=\left\langle \frac{2}{3}t^3, t, t^2\right\rangle, \quad 0\leq t\leq 1.$

Hint: $(a+b)^2=a^2+b^2+2ab$

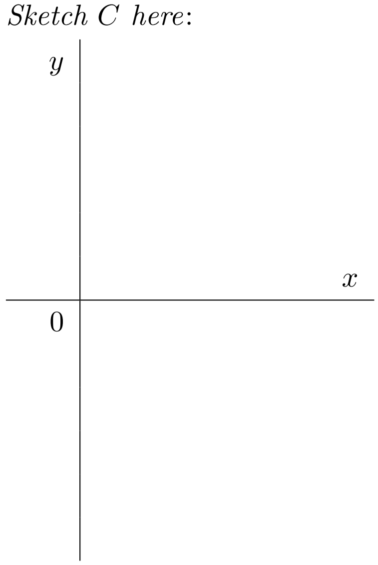
3. Find the work done by the force field $\mathbf{F}(x,y) = \left\langle 5+y, -\frac{1}{3}x \right\rangle$ on a particle that moves along the curve $y = x^3$ from $(-1,-1)$ to the point $(1,1)$.

4. Given the line integral

$$I = \int_C (\sin y + 2011 \sec x^2 + 7y)\,\mathrm{d}x + (x \cos y + e^y + 10x)\,\mathrm{d}y$$

where the path C consists of the line segment from $(0,4)$ to $(0,-1)$, quarter of the circle $x^2+y^2=1$ from $(0,-1)$ to $(1,0)$, and the line segment from $(1,0)$ to $(0,4)$. Use Green's theorem to **evaluate** the given integral and **sketch** the curve C indicating the *positive direction.*

Sketch C here:



5. Let $\vec{F}(x, y) = \langle x^4, y^4, z^4 \rangle$.

a) Show that $\vec{F}$ is conservative vector field.

b) Compute $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$ where C is any path from the point M(0,0,0) to the point N(1,2,-1).

6. Surface S given by $\mathbf{r}(u,v) = \left\langle e^{u-1}, -e^{v}, u+v \right\rangle$, $-5 \le u \le 2$, $0 \le v \le 3$.

(a) Find an equation of the tangent plane at the point $(1,-1,1)$ to S.

(b) Set up, but <u>do not evaluate</u> an (*iterated*) integral for the surface area of S.

7. Let $\mathbf{F}(x, y) = \langle e^{2y}, 2xe^{2y} + 2\rangle$.

(a) Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

(b) Find the work done by the given force field $\mathbf{F}$ in moving an object along the arc of the curve $y = x^2 \sin(\pi - x)$ from the point $A(0, 0)$ to the point $B(\pi, 0)$.

(c) Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the circle $x^2 + y^2 = 2012$. Justify your answer.

8. Find the flux of the vector field $\mathbf{F}(x, y, z) = \left\langle x, y, \cos z^2 \right\rangle$ across S which is the part of the circular cylinder $x^2 + y^2 = 9$ between the planes $z = -2$ and $z = 1$ with outward orientation.

9. Let S be the part of the surface $z = x^2 + y$ that lies above the rectangle $D = \{(x, y)|\ 0 \leq x \leq 1, -1 \leq y \leq 2.\}$ Evaluate $\displaystyle\iint_S 24x\,\mathrm{dS}$.