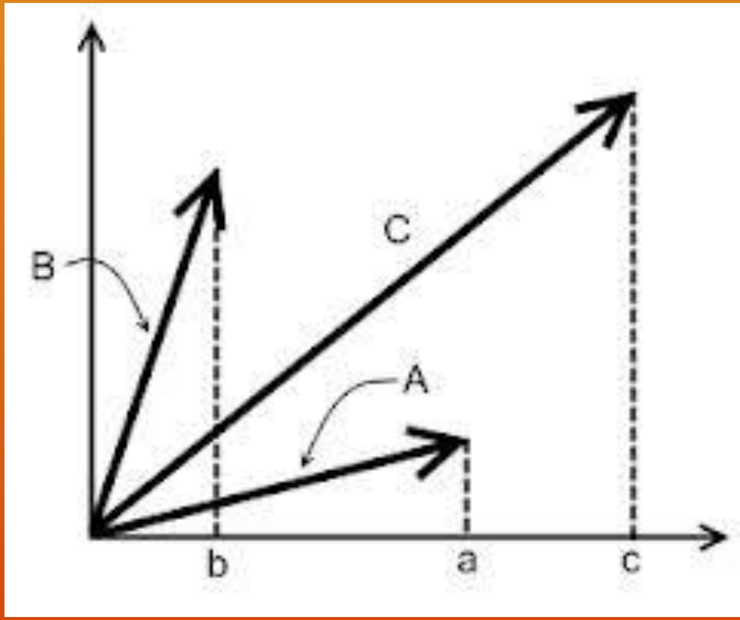
Vectors



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Uses of Vectors

- Projectile motion
- Determine resultant force
  - Construction of buildings
- Determine direction and magnitude
- Describe linear motion
- Momentum
- Work

History of Vectors

- Early mathematicians contributed to the concept of vectors
- Isaac Newton "started the ball rolling" with his theorem:
  - "A body, acted on by two forces simultaneously, will describe the diagonal of a parallelogram in the same time as it would describe the sides by those forces separately."
  - However, he did not have the idea of a vector.
  - He did get come relatively close to an idea that because forces have both direction and magnitude, the forces can be added to produce a new force
- Other mathematicians expanded on Newton's ideas
  - Caspar Wessel, Carl Friedrich Gauss, Jean Robert Argand, John Warren, C.V. Mourey, and William Rowan Hamilton

Newton

Hamilton

Argand

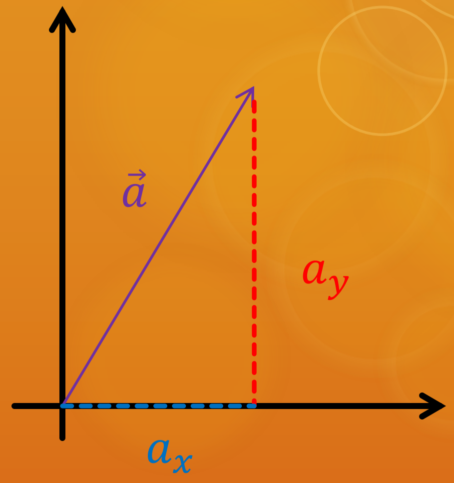
Gauss

Got a Little Story for Ya, Ags!

- The vector was walking down Cartesian drive when he bumped into a confused Scalar.
- The vector asked him what was wrong and he replied, "Help! I have no direction!”
- *Vector* in Latin means *carrier* and it also has that meaning in English.
  - The mosquito is the vector of malaria.

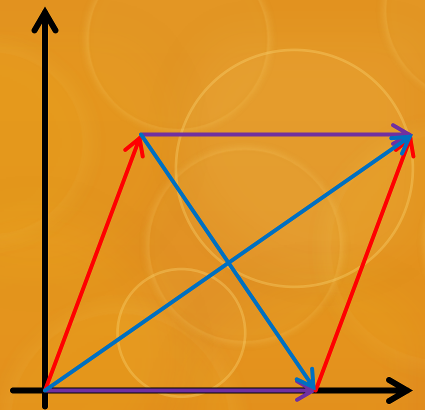
Introduction to Vectors

- What is a vector?
  - Quantity with direction
- Written almost like an ordered pair
  - Vector $\vec{a}$ is written:
    - $\vec{a} = \langle a_x , a_y \rangle$
      - a_x and a_y are components
- Two vectors are equal if their lengths and slopes are the same

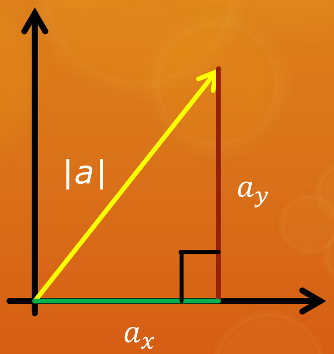


Vector Operations

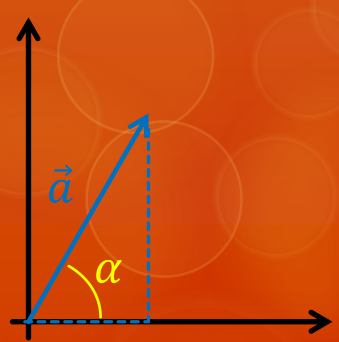
- Addition
  - $\vec{a} + \vec{b}$ = ‹a_1+b_1 , a_2 + b_2› or ‹a_x+b_x , a_y+b_y›
- Subtraction
  - $\vec{a}$ - $\vec{b}$ = ‹a_1 – b_1 , a_2 – b_2› or ‹a_x – b_x , a_y - b_y›
- Multiplication
  - 3 x $\vec{a}$ = ‹3a_1 , 3a_2› or ‹3a_x , 3a_y›
- Multiplication (multiply a vector times a vector)
  - Called the "Dot Product"
  - $\vec{a} \bullet \vec{b}$ = ‹($a_1 b_1$) + ($a_2 b_2$)› or ‹($a_x b_x$) + ($a_y b_y$)›
  - You will get a number in this case, not a vector!
- Magnitude
  - $|a| = \sqrt{a_1{}^2 + a_2{}^2}$ or $\sqrt{a_x{}^2 + a_y{}^2}$
- Unit Vector
  - $\vec{a} = |\vec{a}|$<cos(α), sin(α)>



Parallelogram Rule – Can be used for addition and subtraction

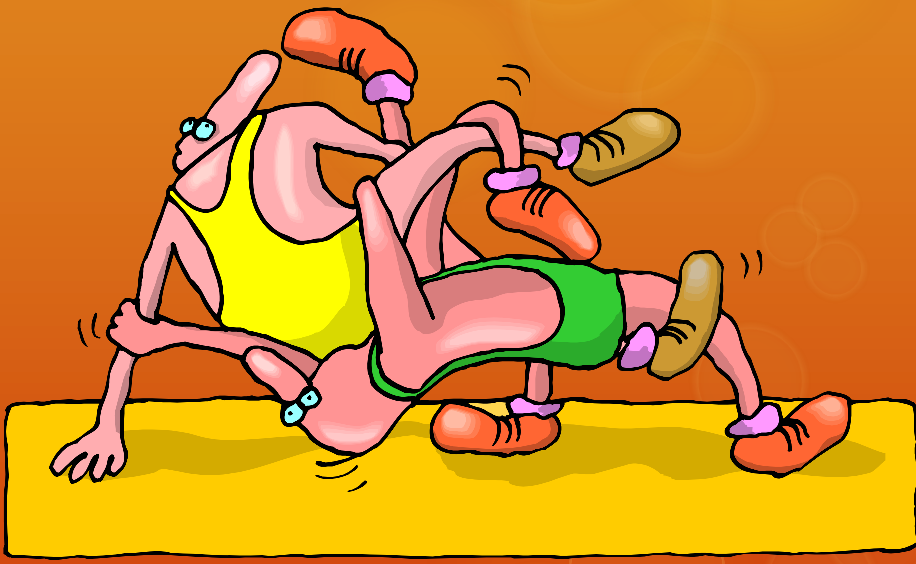


Right Triangle Rule – used to find magnitude of a vector



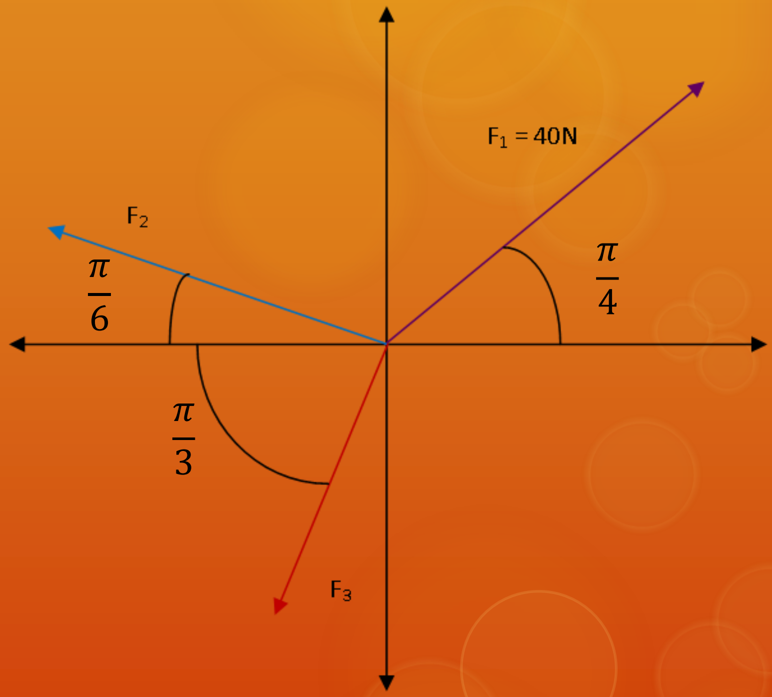
The WWE Championship Fight!

- Three WWE fighters are wrestling over a championship belt.
- The 3 wrestlers are pulling from a point in the middle of the mat at the center. The first wrestler is pulling 45° to the right above center, the second wrestler is pulling 30° to the left above the center, and the third wrestler is pulling 60° to the left below the center. At this point the belt is in equilibrium.
- If the first wrestler pulls with a force of 40 Newtons, with what force are the other two wrestlers pulling?



The Setup

- The best way to solve this problem is by using vectors.
- According to the problem, we introduce a coordinate system, shown to the right.
- We know that at the given point, the belt is in equilibrium. Therefore, the sum of the x-components and the sum of the y-components are equal to zero.
- This gives us the equations:
  - $\sum F_x = F_{1x} + F_{2x} + F_{3x} = 0$
  - $\sum F_y = F_{1y} + F_{2y} + F_{3y} = 0$
- The forces can be written as vectors as follows:
  - $\vec{F_1} = |\vec{F_1}|<\cos(\frac{\pi}{4}), \sin(\frac{\pi}{4})>$
  - $\vec{F_2} = |\vec{F_2}|<\cos(\frac{5\pi}{6}), \sin(\frac{5\pi}{6})>$
  - $\vec{F_3} = |\vec{F_3}|<\cos(\frac{4\pi}{3}), \sin(\frac{4\pi}{3})>$



How to Solve the Problem

- $\vec{F_1} = 40\langle\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle$ $\qquad \vec{F_1} = \langle 20\sqrt{2}, 20\sqrt{2}\rangle$

  $\vec{F_2} = |\vec{F_2}|\langle-\frac{\sqrt{3}}{2}, \frac{1}{2}\rangle$ $\qquad \vec{F_2} = \langle|\vec{F_2}|-\frac{\sqrt{3}}{2}, |\vec{F_2}|\frac{1}{2}\rangle$

  $\vec{F_3} = |\vec{F_3}|\langle-\frac{1}{2}, -\frac{\sqrt{3}}{2}\rangle$ $\qquad \vec{F_3} = \langle|\vec{F_3}|-\frac{1}{2}, |\vec{F_3}|-\frac{\sqrt{3}}{2}\rangle$

- $\langle 20\sqrt{2} + (|\vec{F_2}|-\frac{\sqrt{3}}{2}) + (|\vec{F_3}|-\frac{1}{2}), 20\sqrt{2} + |\vec{F_2}|\frac{1}{2} + (|\vec{F_3}|-\frac{\sqrt{3}}{2})\rangle = \langle 0,0\rangle$
- $20\sqrt{2} + (|\vec{F_2}|-\frac{\sqrt{3}}{2}) + (|\vec{F_3}|-\frac{1}{2}) = 0$ $\qquad 20\sqrt{2} + |\vec{F_2}|\frac{1}{2} + (|\vec{F_3}|-\frac{\sqrt{3}}{2}) = 0$
- $20\sqrt{2} + (|\vec{F_2}|-\frac{\sqrt{3}}{2}) + (|\vec{F_3}|-\frac{1}{2}) = 0$

  $+$

  $20\sqrt{2} + |\vec{F_2}|\frac{1}{2} + (|\vec{F_3}|-\frac{\sqrt{3}}{2}) = 0$

- $\sqrt{3}[20\sqrt{2} + (|\vec{F_2}|\frac{1}{2}) + (|\vec{F_3}|-\frac{\sqrt{3}}{2})] = 0(\sqrt{3})$
- $20\sqrt{6} + (|\vec{F_2}|\frac{\sqrt{3}}{2}) + (|\vec{F_3}|-\frac{3}{2}) = 0$
- $20(\sqrt{2} + \sqrt{6}) - 2|\vec{F_3}| = 0$
- $20(\sqrt{2} + \sqrt{6}) = 2|\vec{F_3}|$
- $|\vec{F_3}| = 10(\sqrt{2} + \sqrt{6}) \approx 38.637$ N

- 1. Insert the values given by the problem. Simplify.
- 2. Break up the equations into components.
- 3. Set x components equal to 0. Set y components equal to 0.
- 4. Solve for either $|\vec{F_2}|$ or $|\vec{F_3}|$. Use the equations in step 3.
- 5. We solve for $|\vec{F_3}|$ first.

The Last Step

- We substituted the value of $|\overrightarrow{F_3}|$ that we obtained into the first equation, giving us the equation:

$$20\sqrt{2} + (|\overrightarrow{F_2}|\text{-}\tfrac{\sqrt{3}}{2}) + (|\overrightarrow{F_3}|\text{-}\tfrac{1}{2}) = 0$$

$$20\sqrt{2} + (|\overrightarrow{F_2}|\text{-}\tfrac{\sqrt{3}}{2}) + (\text{-}\tfrac{1}{2}[10(\sqrt{2} + \sqrt{6}]) = 0$$

- We then solve this equation for $|\overrightarrow{F_2}|$.

$$20\sqrt{2} + (|\overrightarrow{F_2}|\text{-}\tfrac{\sqrt{3}}{2}) + (\text{-}5\sqrt{2} - 5\sqrt{6}) = 0$$

$$(|\overrightarrow{F_2}|\tfrac{\sqrt{3}}{2}) = 20\sqrt{2} - 5\sqrt{2} - 5\sqrt{6}$$

$$(|\overrightarrow{F_2}|\tfrac{\sqrt{3}}{2}) = 15\sqrt{2} - 5\sqrt{6}$$

$$\tfrac{\sqrt{3}}{2}(|\overrightarrow{F_2}|\tfrac{\sqrt{3}}{2}) = \tfrac{\sqrt{3}}{2}(15\sqrt{2} - 5\sqrt{6})$$

$$|\overrightarrow{F_2}| = \frac{30\sqrt{2}}{\sqrt{3}} - \frac{10\sqrt{6}}{\sqrt{3}} \approx 10.353 \text{ N}$$

- Conclusion: Wrestler 2 is pulling with a force of 10.353 N. Wrestler 3 is pulling with a force 38.637 N.

Babysitting

- Kim is babysitting for the McDonalds. Since the children have just come home from a birthday party, they insist that Kim push them across the room in their new box car.
- A) How much work is done by Kim if she pushes the box by a force of 60 lb acting in the direction 30° below the horizontal as she pushes the children 15 feet across the room?
- B) If Kim pulls with the same force over the same distance, but at 30° above the horizontal, does she do the same amount of work?

Setting up the Problem

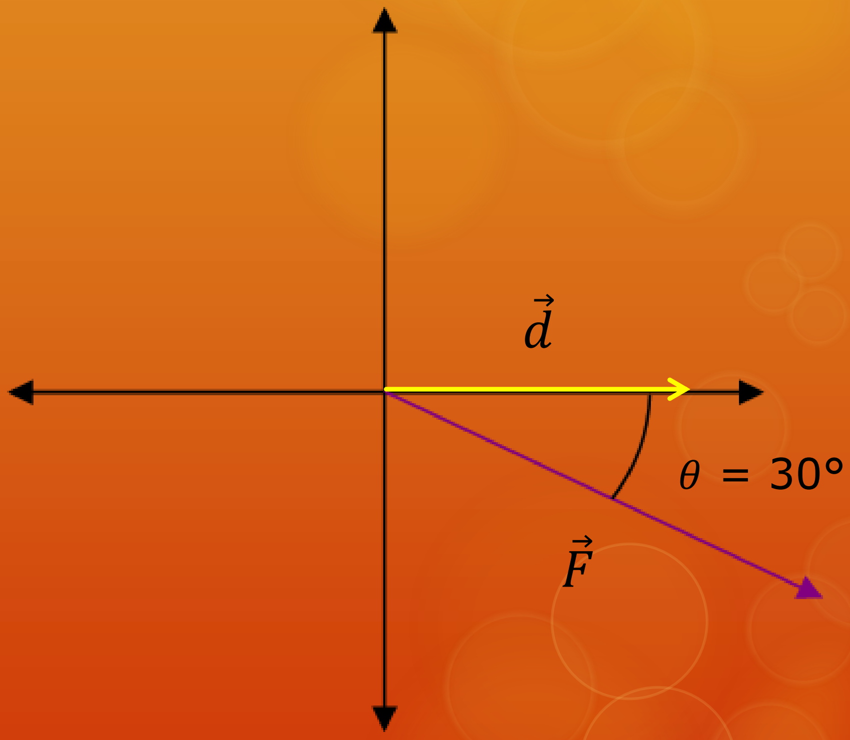
- We know that work is the dot product of the force vector and distance vector.

$$W = \vec{F} \bullet \vec{d}$$

- However, this equation is only for motion in a straight line motion. Kim applies a force at an angle, not in a straight line. Therefore, we must use the following equation to determine the straight line displacement.

$$W = |\vec{F}| \ x \ |\vec{d}| cos(\theta)$$

- This equation will give us the work done when a force is applied at an angle.



Solving the Problem

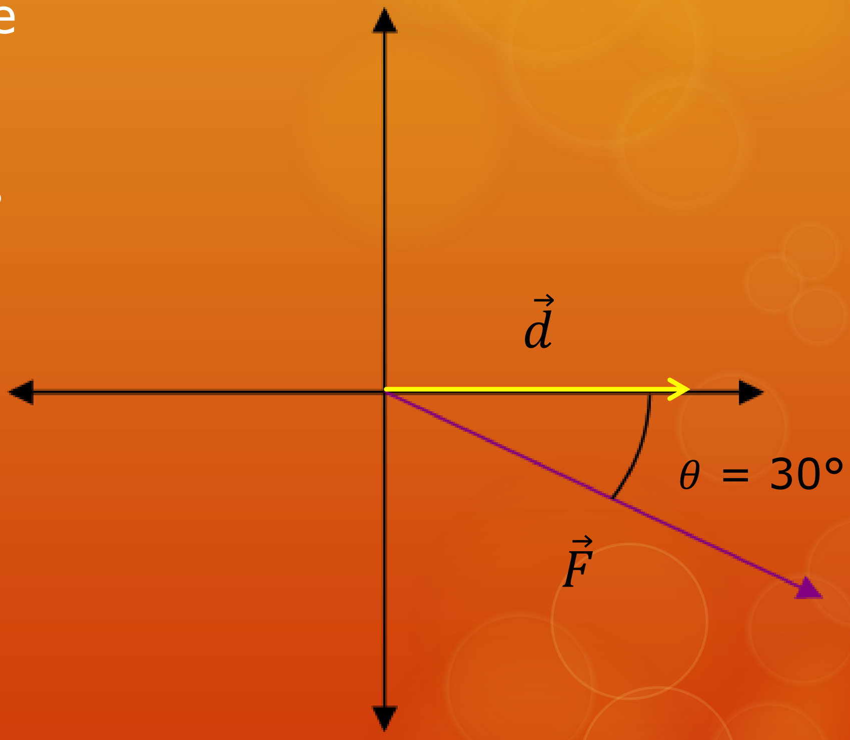
- In order to solve this problem, we simply multiply the magnitude of the force by the straight-line displacement. Our equation looks like this:

$$W = (60 \text{ lbs})(15 \text{ ft})(\cos(\tfrac{\pi}{6}))$$

$$W = (60 \text{ lbs})(15 \text{ ft})(\tfrac{\sqrt{3}}{2})$$

$$W = 450\sqrt{3} \text{ ft}\bullet\text{lb}$$

$$W \approx 779.423 \text{ ft}\bullet\text{lb}$$



Solving the Problem

- To solve part B, we can use

$$W = \vec{F} \bullet \vec{d}$$

- We can again use the same set-up for this problem

$$W = |\vec{F}| \bullet |\vec{d}| cos(\theta)$$

- Since Kim is pulling with the same force and distance as in part A and over the same distance, the answer we obtain for work will be the same

$$W = (60 \text{ lbs})(15 \text{ ft})(\cos(\frac{\pi}{6}))$$

$$W = (60 \text{ lbs})(15 \text{ ft})(\frac{\sqrt{3}}{2})$$

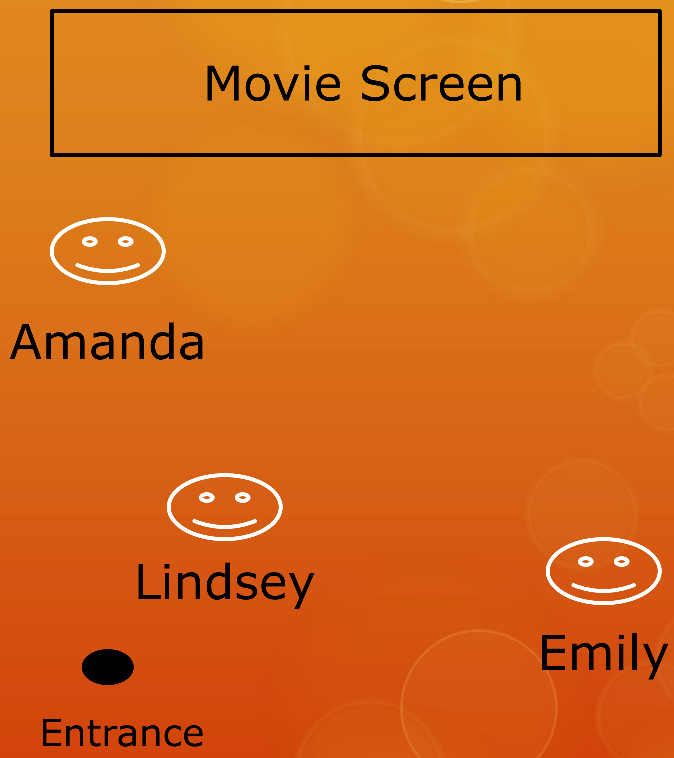
$$W = 450\sqrt{3} \text{ ft} \bullet \text{lb}$$

$$W \approx 779.423 \text{ ft} \bullet \text{lb}$$

- This is because work is a scalar quantity, not a vector. Therefore, direction that the force is applied doesn't make a difference, as long as the force is the same and over the same distance.

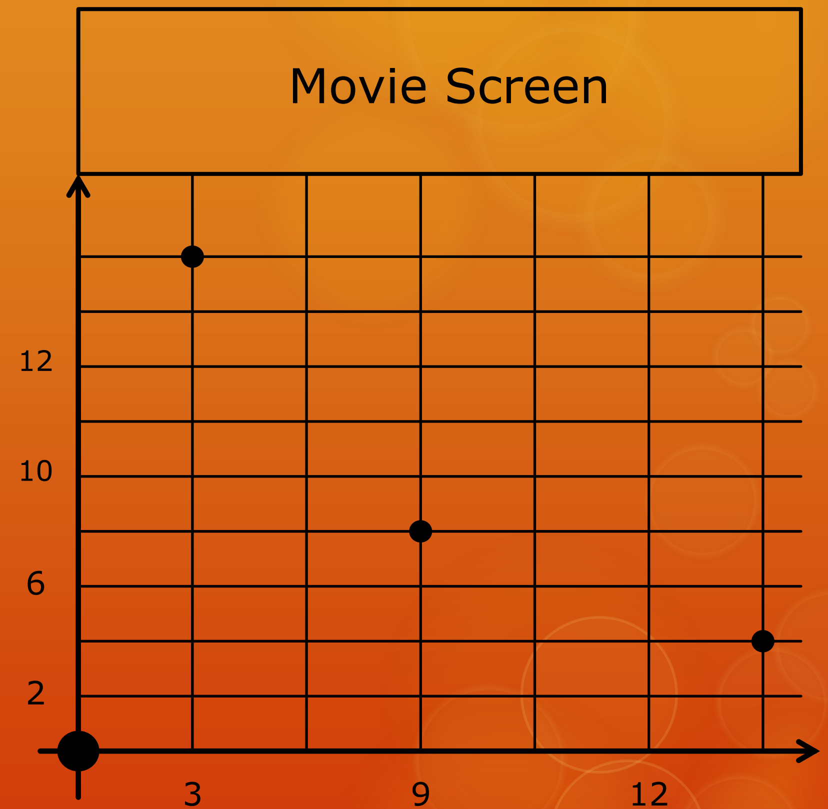
The Movie Date

- Amanda, Lindsey, and Emily are all at the local movie theatre. However, because the movie is a new release, they could not find three seats together.
- Billy also decides to go watch the movie. Upon entering the theatre, he sees Amanda, Lindsey, and Emily. Unbeknownst to the three girls, Billy has a crush on all of them.
  - Part A: Billy wants to be nice so he decides to get popcorn and share it amongst the girls. What is the shortest path he can take? (Billy will enter the movie theatre from the entrance)
  - Part B: Lindsey also has a secret crush on Billy. In what seat(s) could she sit so that Billy doesn't have to walk so far?



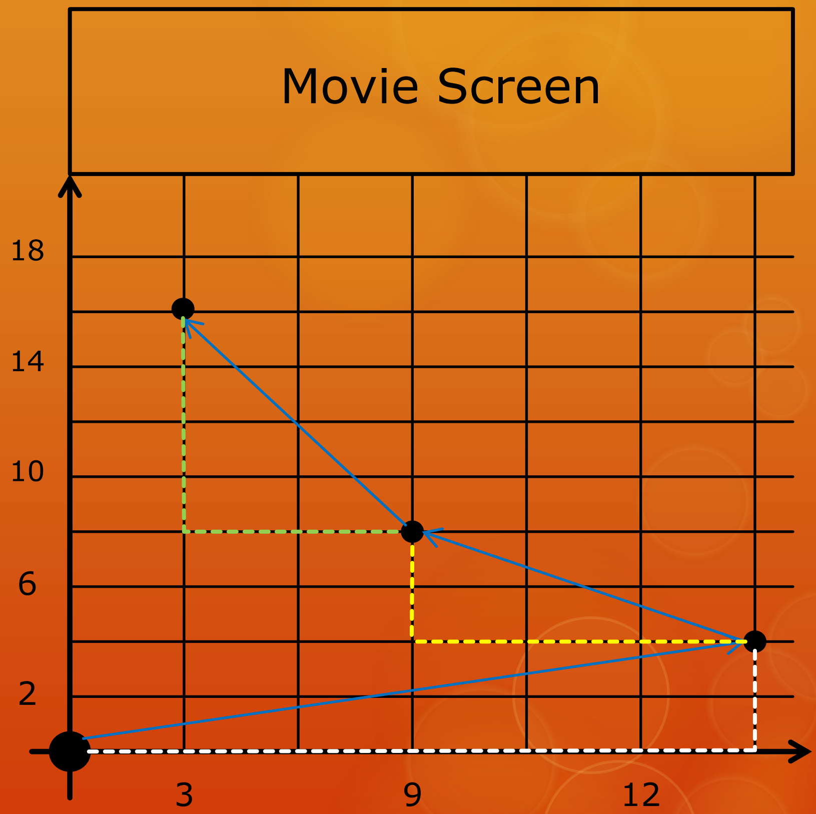
The Approach to the Problem

- The best way to approach this problem is by vectors. We will first introduce a coordinate system to express the girls position's graphically. (We have replaced the smiley faces with points for clarification purposes). The x-axis represents the row number and the y-axis represents the seat number.
- The easiest way to set up this coordinate system is by placing the entrance to the movie theatre at the origin.
- We place the origin at the entrance because we need to find the distance Billy will travel. Billy will enter from the entrance, so it makes sense to place the origin here.
- Each row is one meter wide, and each seat is one meter long.



Solving the Problem

- Part A of the problem asks us to find the shortest path that Billy can travel when giving popcorn to the three girls.
- We will use vectors to solve this problem, as this is the easiest way.
- In order to travel the least distance, Billy should walk the path represented by the blue vectors.
- We will use components to find Billy's path (the blue vectors).
- We first "break" each vector into components. This will make finding the magnitude much easier.
  - The components are represented by the dashed lines.



Solving the Problem

- We have removed the grid so that our work is easier to read.
- We will find the magnitude of the first vector by using the first triangle.

$$c^2 = 4^2 + 15^2$$

$$c = \sqrt{241}$$

- Now, we solve for the magnitude of the second triangle.

$$d^2 = (-6)^2 + (4)^2$$

$$d = \sqrt{52}$$

- Finally, we find the magnitude of the third vector.

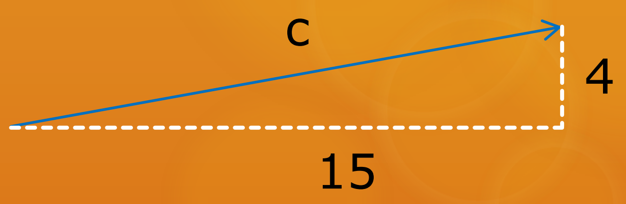
$$e^2 = (-6)^2 + 8^2$$

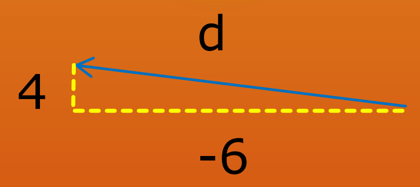
$$e = 10$$

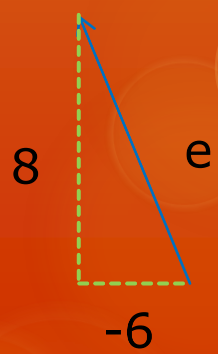
- We now add these magnitudes together to get the total distance that Billy traveled.

$$c + d + e$$

$$\sqrt{241} + \sqrt{52} + 10 \approx 32.74 \text{ meters}$$





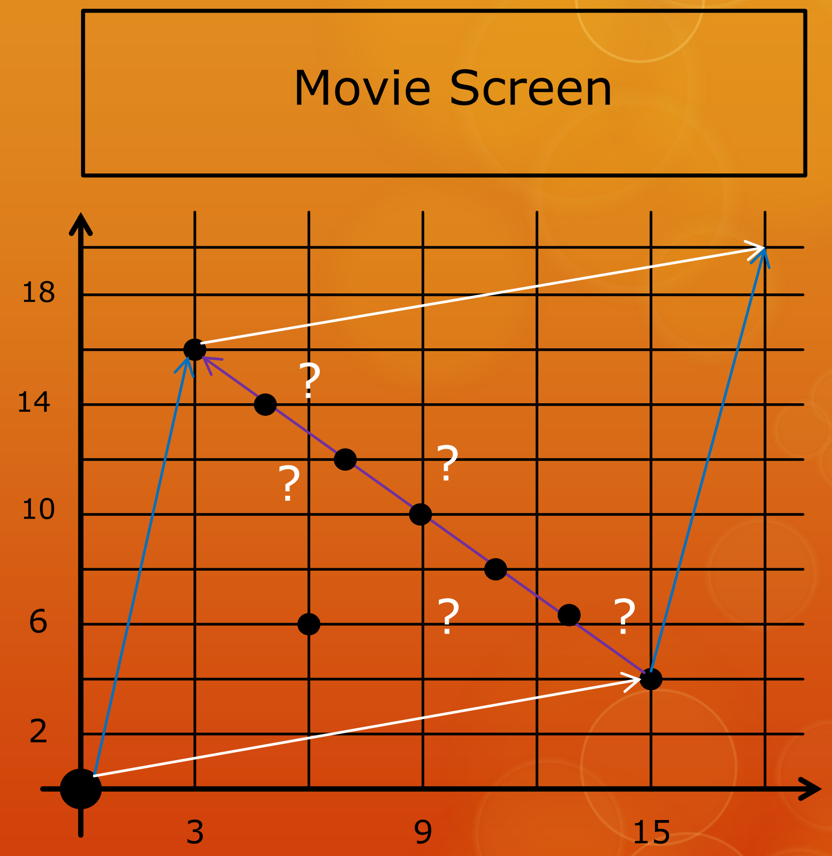


Part B

- Part B of this problem asks us to find where Lindsey should sit so that Billy can walk an even shorter distance to give all three girls popcorn.
- We will use the Parallelogram Law to solve this problem.
- First, we should find the vector between Emily and Amanda.

<3,16> - <15, 4>

<-12, 12>

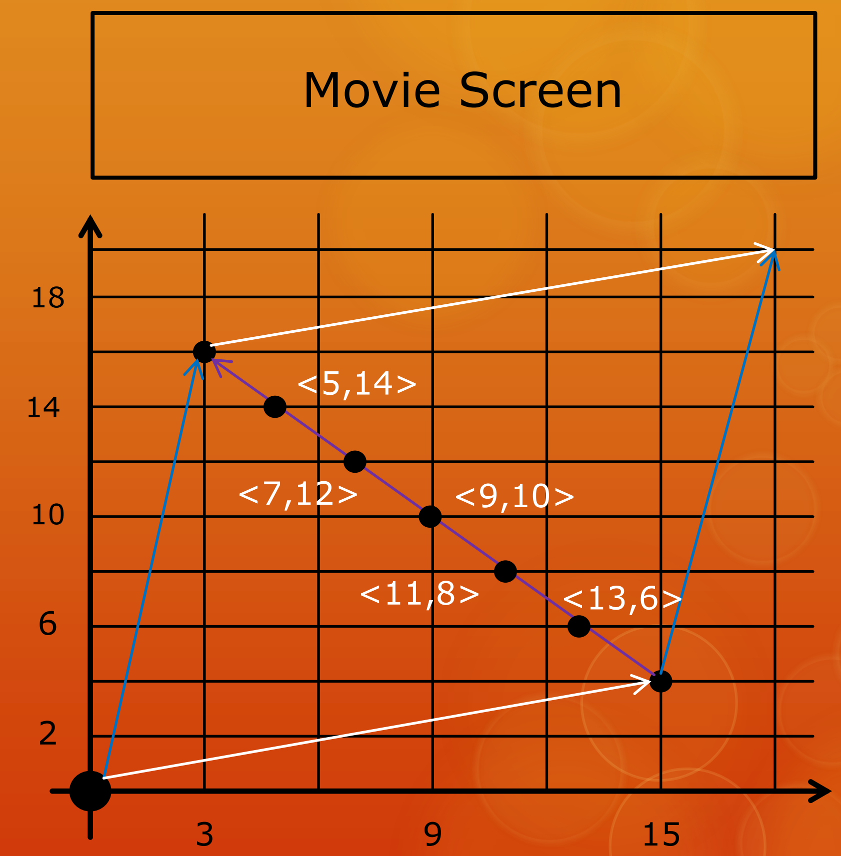


The Last Step

- Technically, there are infinitely many points along the vector <-12, 12> that Lindsey could sit.
- However, since Lindsey is sitting in a movie theatre and can not sit in between seats or rows, we will only use whole numbers.
- The slope of a vector is found by the equation:

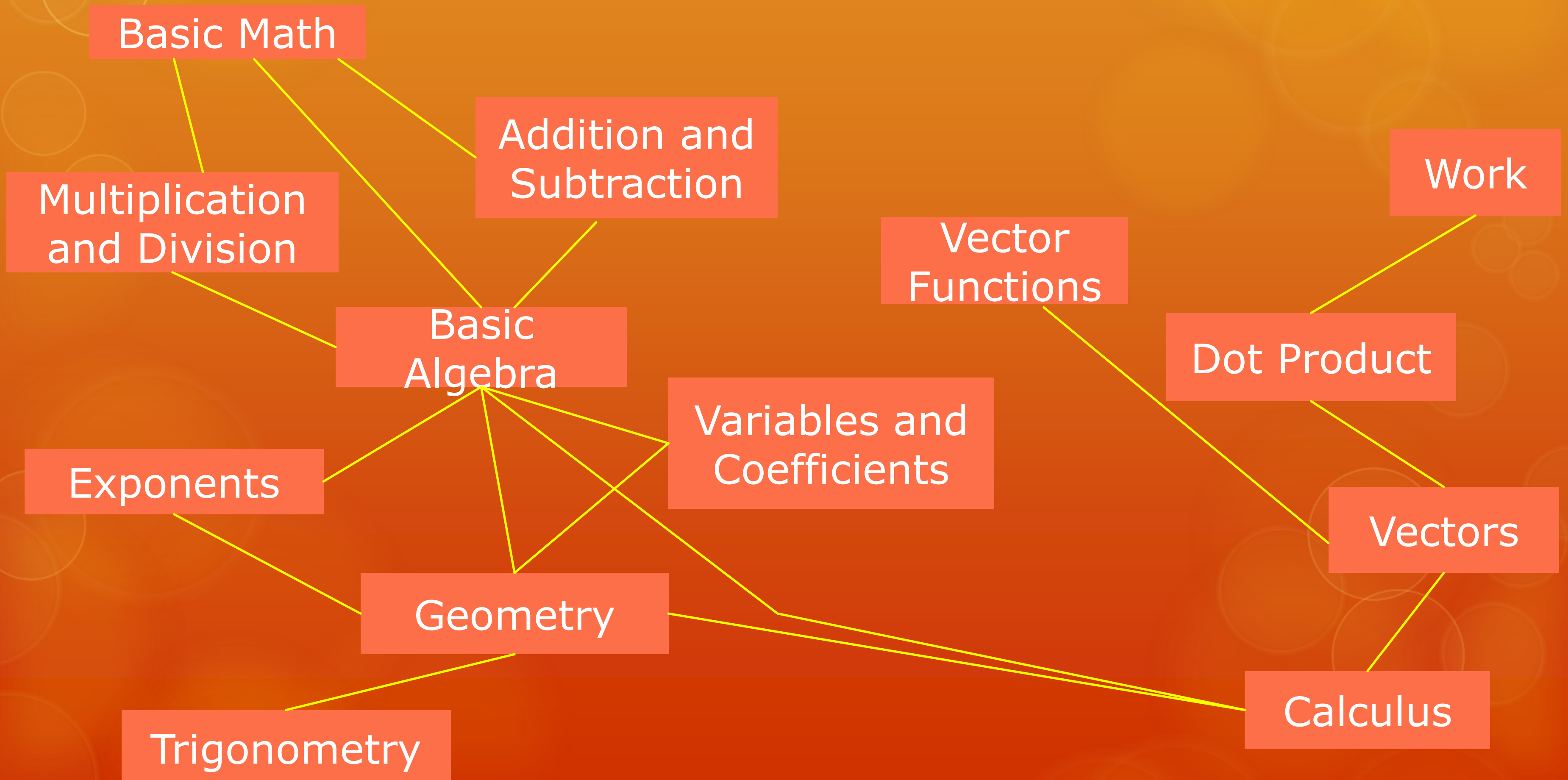
$$\frac{\Delta y}{\Delta x}$$

- In this problem, the slope is -1.
- Some of the places Lindsey could sit are on the diagram.



The Visual Map

- In this PowerPoint, we have demonstrated multiple concepts from basic math through calculus.



Bibliography

- http://jokes4all.net/vectors.html
- https://www.dpmms.cam.ac.uk/~kf262/MMM/L2/L2_2.pdf
- We used Google Images for the various pictures on the slides.
- We also used Dr. Oskana Shatalov's notes for vector formulas and operations.