Homework Assignment 5 in Differential Equations, MATH308

due March 7, 2012

Topics covered : *method of variation of parameters; mechanical and electrical vibrations; forced vibration (corresponds to sections 3.6, 3.7, and 3.8 in the textbook); symbolic solutions of ODE's and plot of solutions using MATLAB (commands* dsolve *and* ezplot*) ; use that the graviational acceleration* $g = 32\frac{lb \cdot ft}{s^2}$

1. Use the method of variation of parameter to find the general solution of the given differential equation:

   (a) $y'' - 3y' - 10y = 4e^{5t}$;

   (b) $y'' - 2y' + y = \dfrac{e^t}{t}, \quad t > 0.$

2. (a) Determine ω_0, R and δ so as to write the expression $\cos 4t - \sqrt{3}\sin 4t$ in the form $R\cos(\omega_0 t - \delta)$;

   (b) Write the expression $\cos 5t - \cos 2t$ as a product of two trigonometric functions of different frequencies.

3. A mass weigh 8 lb stretches a spring 6 in.

   (a) Assume that there is no damping. If the mass pulled up 3 in and then released with no initial velocity, determine the position u of the mass at any time t. Find the frequency, period, and amplitude of the motion.

   (b) Assume that there is damping and we can change the damping constant. What is the critical damping constant?

4. A spring is stretched 3 in by a mass that weighs 2 lb. The mass is attached to a dashpot mechanism that has a damping constant of $2\frac{lb \cdot s}{ft}$ and is acted on by an external force of $2\cos 3t$ lb.

   (a) Determine the steady state solution of this system;

   (b) If the external force is $2cos\,\omega t$ determine the frequency $\omega > 0$ for which the amplitude of the steady state solution is maximal.

5. Consider the initial value problem

$$u'' + 100u = 3\cos 9t, \quad u(0) = 0, \quad u'(0) = 0$$

   (a) Solve this initial value problem without using computer;

   (b) Solve the same initial value problem with MatLab using the command dsolve (print and attach the program and the result of the computations);

   (c) Plot the solution of this initial value problem using MatLab (try to choose a nice interval [0,T] in which to plot the graph so that the effect of amplitude modulation will be seen).