Homework Assignment 6 in Differential Equations, MATH308

due March 21, 2012

Topics covered : *definition and properties of Laplace transform; inverse Laplace transform of rational functions; solution of initial value problems using Laplace transform; step function and Laplace transform of discontinuous functions (corresponds to sections 6.1, 6.2, 6.3 in the textbook)*

1. Recall that the hyperbolic cosine $\cosh t$ and hyperbolic sine $\sinh t$ are defined as follows:

$$\cosh t = \frac{e^t + e^{-t}}{2}, \quad \sinh t = \frac{e^t - e^{-t}}{2}.$$

   Using the definition of the Laplace transform, find the Laplace transform of the given function (below a and b are real constants):

   (a) $f(t) = \sinh bt$;

   (b) $f(t) = e^{at} \cosh bt$

   (show your work).

2. Find the inverse Laplace transform of the given function:

   (a) $F(s) = \dfrac{3s}{s^2 + 2s - 8}$;

   (b) $F(s) = \dfrac{2s + 5}{s^2 + 6s + 25}$

3. Solve for $Y(s)$, the Laplace transform of the solution $y(t)$ to the given initial value problem (you do not need to find the solution $y(t)$ itself here):

   (a) $y'' - 3y' + 2y = \cos t, \quad y(0) = 0,\ y'(0) = -1$;

   (b) $y'' + y' - y = t^3, \quad y(0) = 1,\ y'(0) = 0$

4. Using the method of Laplace transform solve the following initial value problem:

$$y'' + 6y' + 5y = 12e^t, \quad y(0) = -1,\ y'(0) = 7.$$

5. (a) Find the Laplace transform of the function

$$f(t) = \begin{cases} 0 & t < 1, \\ t & 1 \le t < 2, \\ 1 & 2 \le t. \end{cases}$$

   (b) Find the inverse Laplace transform of the function $\dfrac{e^{-2s} - 3e^{-4s}}{s+2}$.