Homework Assignment 8 in Differential Equations, MATH308

due April 11, 2012 *Sections covered 7.1-7.5*

1. Let $A = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$. Compute $AB - BA$.

2. Transform the given equation into a system of first order differential equations:
   (a) $u'' + 3u' + 4u = \cos t$
   (b) $y^{(3)} - 2y' + y = 0$

3. Express the given system of linear differential equations in matrix form:
   (a) $\begin{cases} x_1' &= 2x_1 - 3x_3 \\ x_2' &= x_2 + 4x_3 \\ x_3' &= x_1 + x_3 \end{cases}$ (b) $\begin{cases} x' &= (\sin t)x + e^t y + \cos t \\ y' &= (\cos t)x - e^t y \end{cases}$

4. Determine whether the following solutions of the the system $x'(t) = Ax(t)$ form a fundamental set of its solutions. If they do, give a general solution of the system.
   (a) $x^1 = e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $x^2 = e^{2t}\begin{pmatrix} -2 \\ 4 \end{pmatrix}$
   (b) $x^1 = \begin{pmatrix} e^t \\ e^t \\ e^t \end{pmatrix}$, $x^2 = \begin{pmatrix} \sin t \\ \cos t \\ -\sin t \end{pmatrix}$, $x^3 = \begin{pmatrix} -\cos t \\ \sin t \\ \cos t \end{pmatrix}$

5. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= x_1 + 3x_2 \\ x_2' &= 12x_1 + x_2 \end{cases} \tag{1}$$

   (a) Find the general solution of the system (1).
   (b) Find the solution of the the system (1) satisfying the initial conditions: $x_1(0) = 1$, $x_2(0) = 1$.
   (c) Find all α_1 and α_2 such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is the solution of of the system (1) with initial condition $x(0) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ then $x(t) \to 0$ as $t \to \infty$.
   (d) Find all β_1 and β_2 such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is the solution of of the system (1) with initial condition $x(0) = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$ then $x(t) \to 0$ as $t \to -\infty$.

6. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= x_1 + 2x_2 + 2x_3 \\ x_2' &= 2x_1 + 3x_3 \\ x_3' &= 2x_1 + 3x_2 \end{cases} \tag{2}$$

   (a) Find the general solution of the system (2).
   (b) Find the solution of the the system (2) satisfying the initial condition $\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$