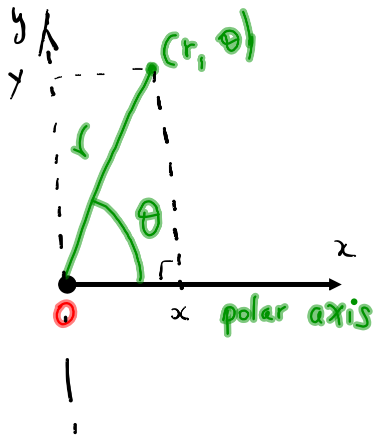
13.4: Polar Coordinates



REVIEW:

The connection between polar and Cartesian coordinates:

$\cos\theta = \dfrac{x}{r}$ $\qquad$ $\sin\theta = \dfrac{y}{r}$

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2+y^2}$ $\qquad$ $\tan\theta = \dfrac{y}{x}$

REMARK 1. In converting from the Cartesian to polar coordinates we must choose θ so that the point (r, θ) lies in the correct quadrant.

$r \geq 0 \qquad 0 \leq \theta < 2\pi$

EXAMPLE 2. *What curve is represented by the polar equation*

(a) $r = 12$

$r = \sqrt{x^2+y^2} = 12 \Rightarrow x^2+y^2 = 12^2$

circle centered at origin with radius 12

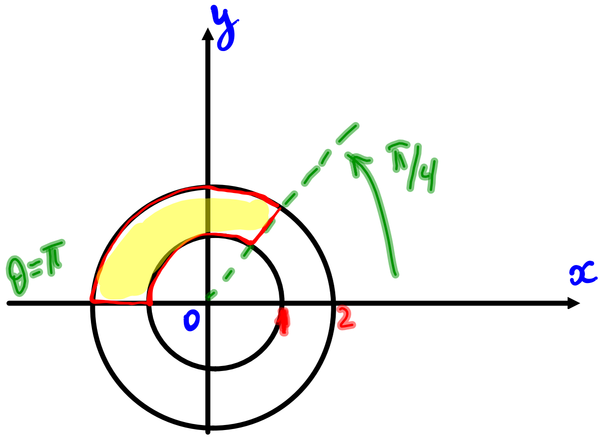
(b) $\theta = \frac{\pi}{3}$

ray with slope π/3

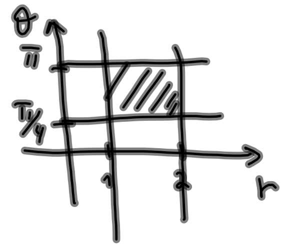
EXAMPLE 3. *Sketch the region in the Cartesian plane consisting of points whose polar coordinates satisfy the following conditions:* $1 \le r \le 2, \quad \pi/4 \le \theta \le \pi.$

$r = 1 \implies x^2 + y^2 = 1$

$r = 2 \implies x^2 + y^2 = 2^2$



compare with



EXAMPLE 4. *Find a polar equation for the curve represented by the given Cartesian equation:*

(a) $x^2 + y^2 = 2by$

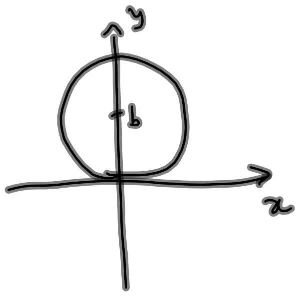
$x = r\cos\theta \,, \quad y = r\sin\theta$ $\Big\}$ $\quad r^2 = 2br\sin\theta$

$$\boxed{r = 2b\sin\theta}$$

Note that this is circle

$$x^2 + \underbrace{y^2 - 2by + b^2}_{(y-b)^2} - b^2 = 0$$

$$x^2 + (y-b)^2 = b^2$$



(b) $(x-a)^2 + y^2 = a^2$

$$x^2 - 2ax + \cancel{a^2} + y^2 = \cancel{a^2}$$

$$x^2 + y^2 = 2ax \Rightarrow r^2 = 2ar\cos\theta \Rightarrow \boxed{r = 2a\cos\theta}$$