5.1: What does $f'(x)$ say about f?

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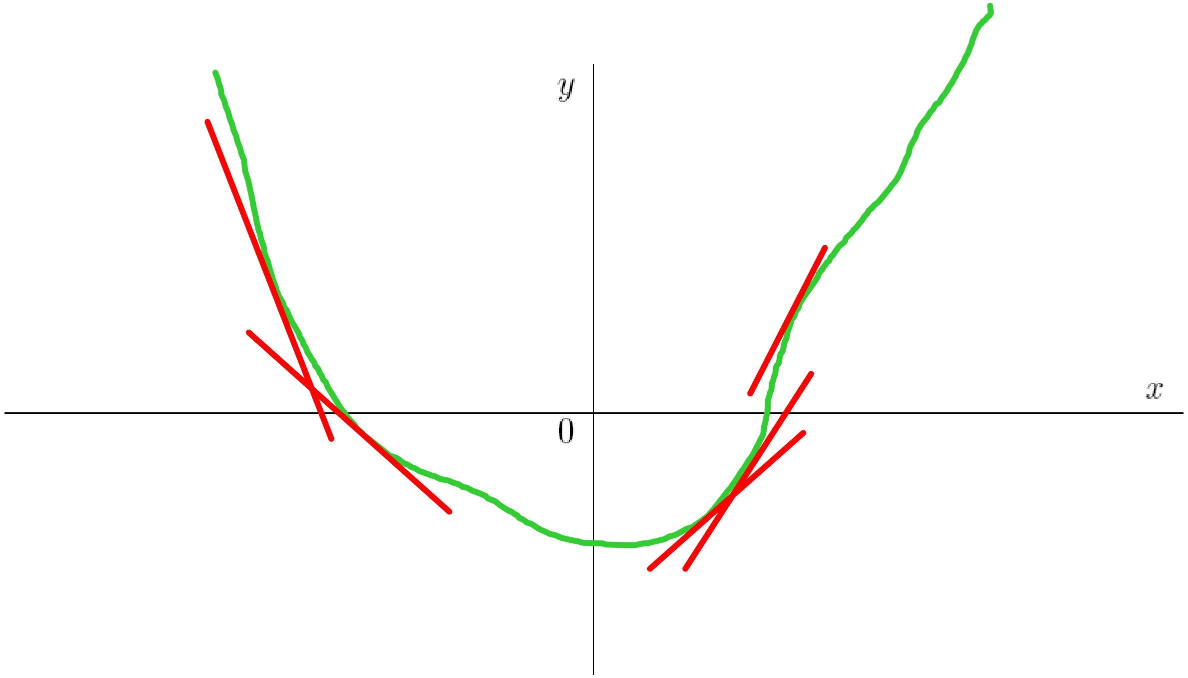
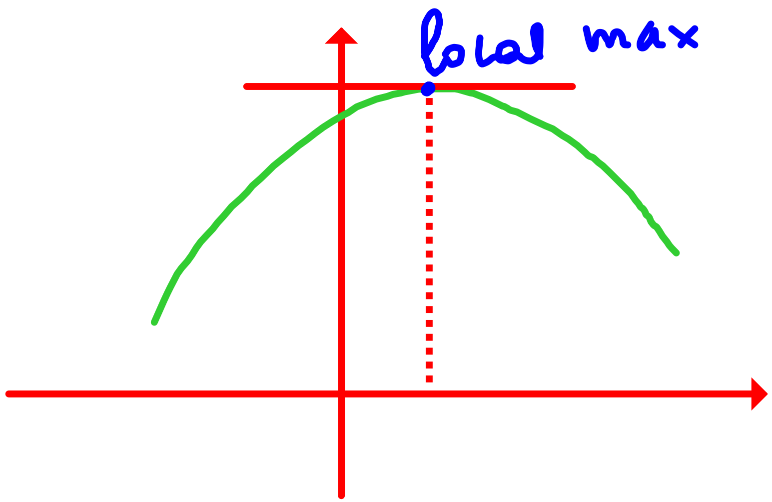
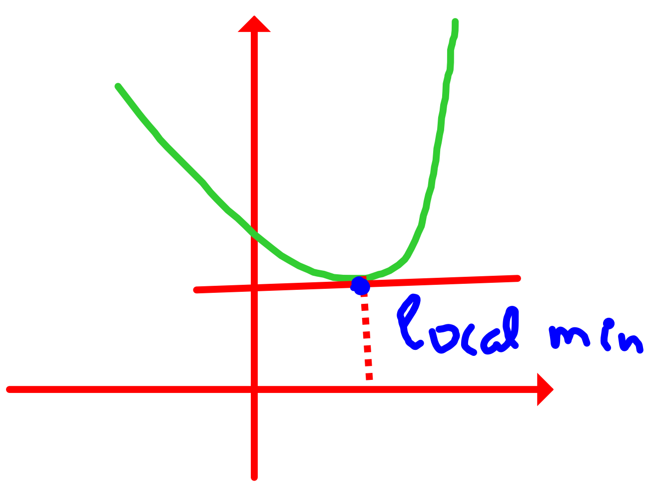


- If $f'(x) > 0$ on an interval, then f is increasing on that interval.
- If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

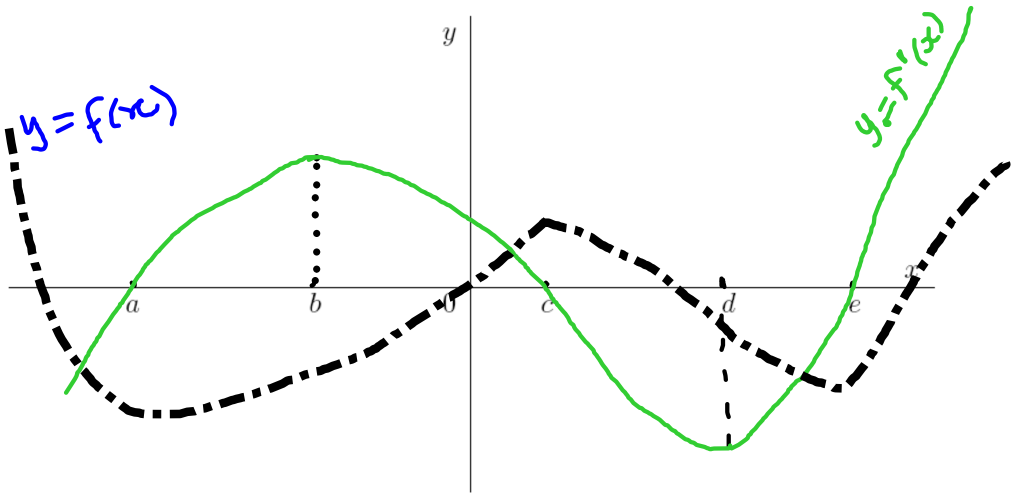
Let $x = a$ be in the domain of $f(x)$.

- . If the derivative $f'(x)$ goes from positive to negative at $x = a$, then the function f has a *local maximum* at $x = a$.
- If the derivative $f'(x)$ goes from negative to positive at x = a, then the function f has a *local minimum* at $x = a$.





EXAMPLE 1. *Consider the graph of the* **derivative** $f'(x)$ *of some function* f.



Answer the following questions:

(a) *Over what intervals is* f *increasing?* (a,c) ∪ (e,∞)

(b) *Over what intervals is* f *decreasing?* (-∞,a) ∪ (c,e)

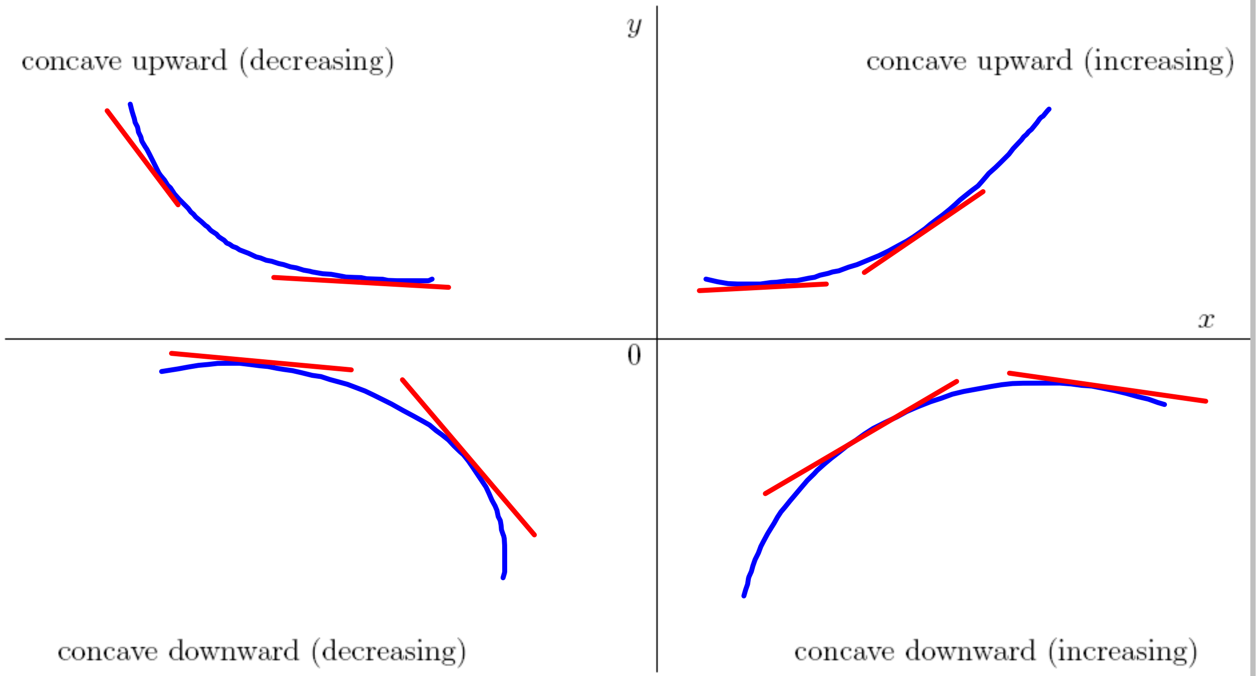
(c) *Determine the* x*-values of* $f(x)$ *that have a horizontal tangent:* x = a, c, e

(d) *Determine the local maximum point(s) of* f*:* x = c

(e) *Determine the local minimum point(s) of* f*:* x = a, e

(f) *Given that* $f(0) = 0$, *sketch a possible graph of* f. see above ↑

Concavity:

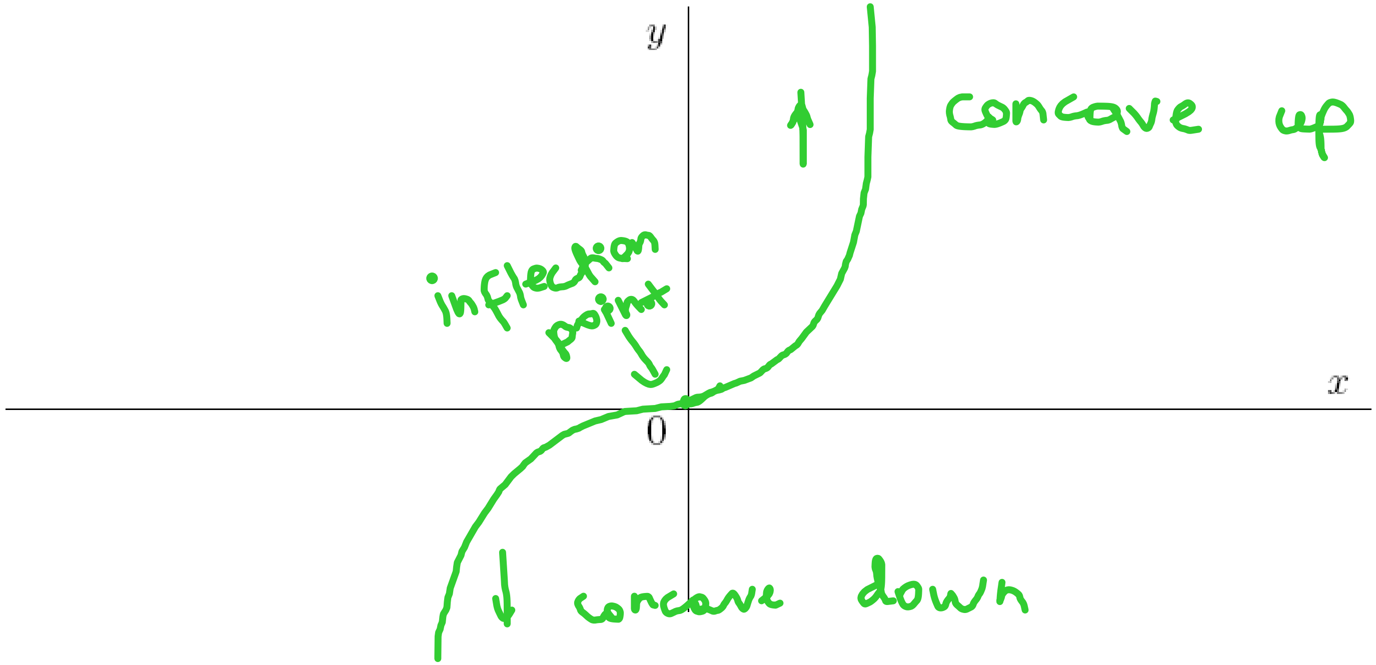


- f is *concave upward* on an interval if all of the tangents to the graph of f on that interval are below the graph of f.
- f is *concave downward* on an interval if all of the tangents to the graph of f on that interval are above the graph of f.

Also,

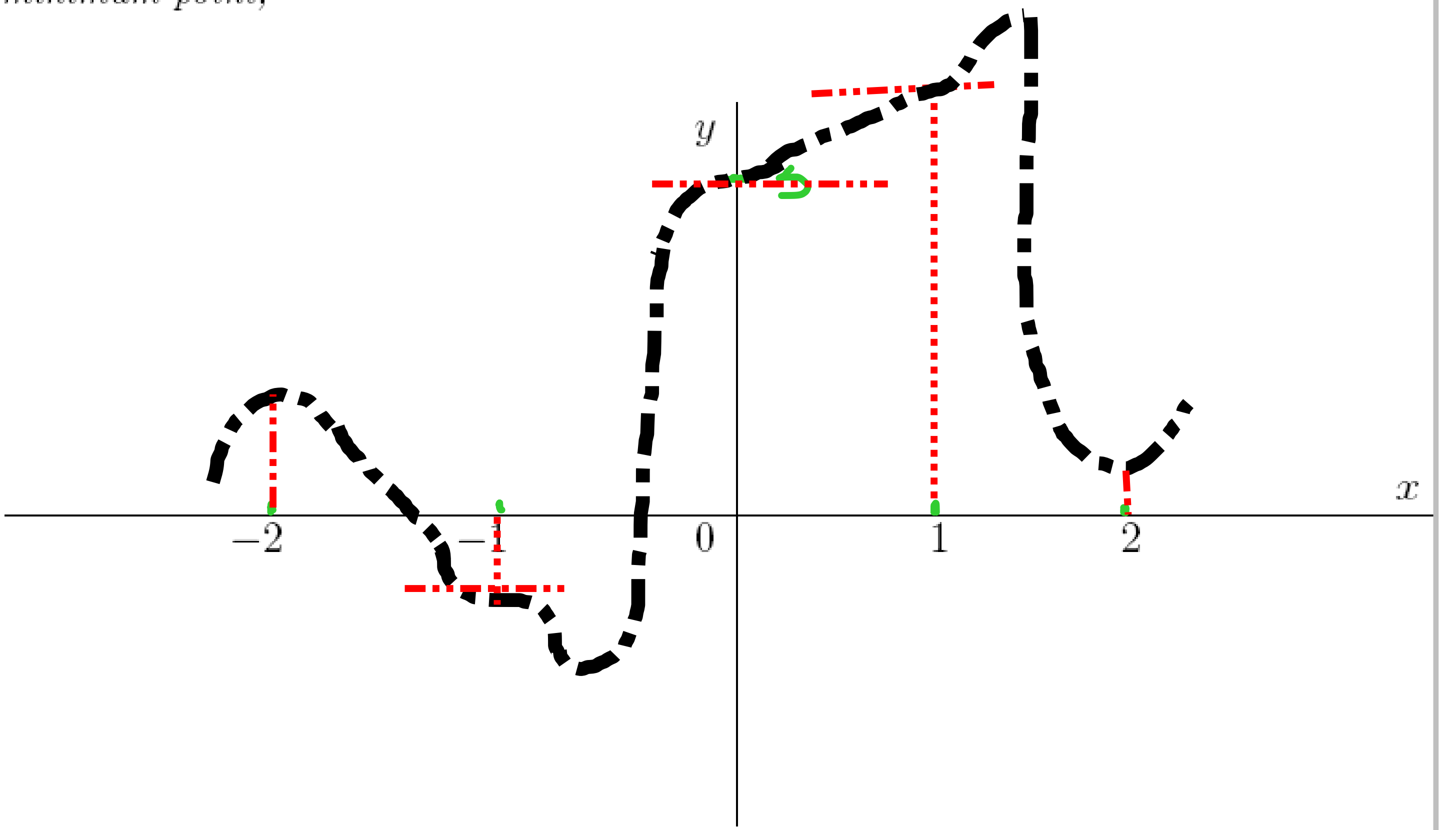
- If the slopes of a curve become progressively larger as x increases, then f is *concave upward.*
- If the slopes of a curve become progressively smaller as x increases, then we say f is *concave downward.*

DEFINITION 2. *If f changes concavity at $x = a$, and $x = a$ is in the domain of f, then $x = a$ is an* **inflection point** *of f.*



EXAMPLE 3. *Sketch a possible graph of a function f that satisfies the following conditions:*

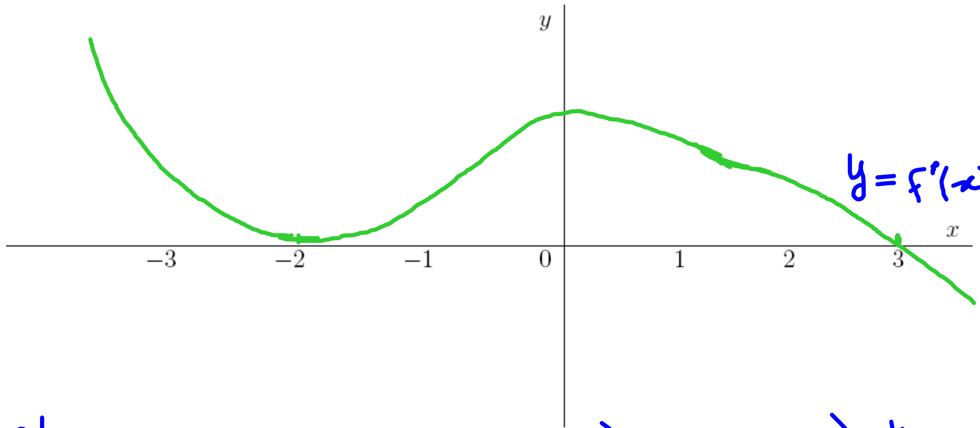
- $f(x)$ *is concave up on* $(-1, 0)$ *and* $(1, \infty)$;
- $f(x)$ *is concave down on* $(-\infty, 1)$ *and* $(0, 1)$;
- $x = 0, \pm 1$ *are the inflection points;*
- $x = -2$ *is the local maximum point;*
- $x = 2$ *is the local minimum point;*
- $f(0) = 3$.



What does f'' say about f?

- If $f''(x) > 0$ for all x on an interval, then f is concave up on that interval.
- If $f''(x) < 0$ for all x on an interval, then f is concave down on that interval.

EXAMPLE 4. *Given the graph of the derivative, $f'(x)$, for some function f. Determine the intervals of concavity and inflection point(s).*

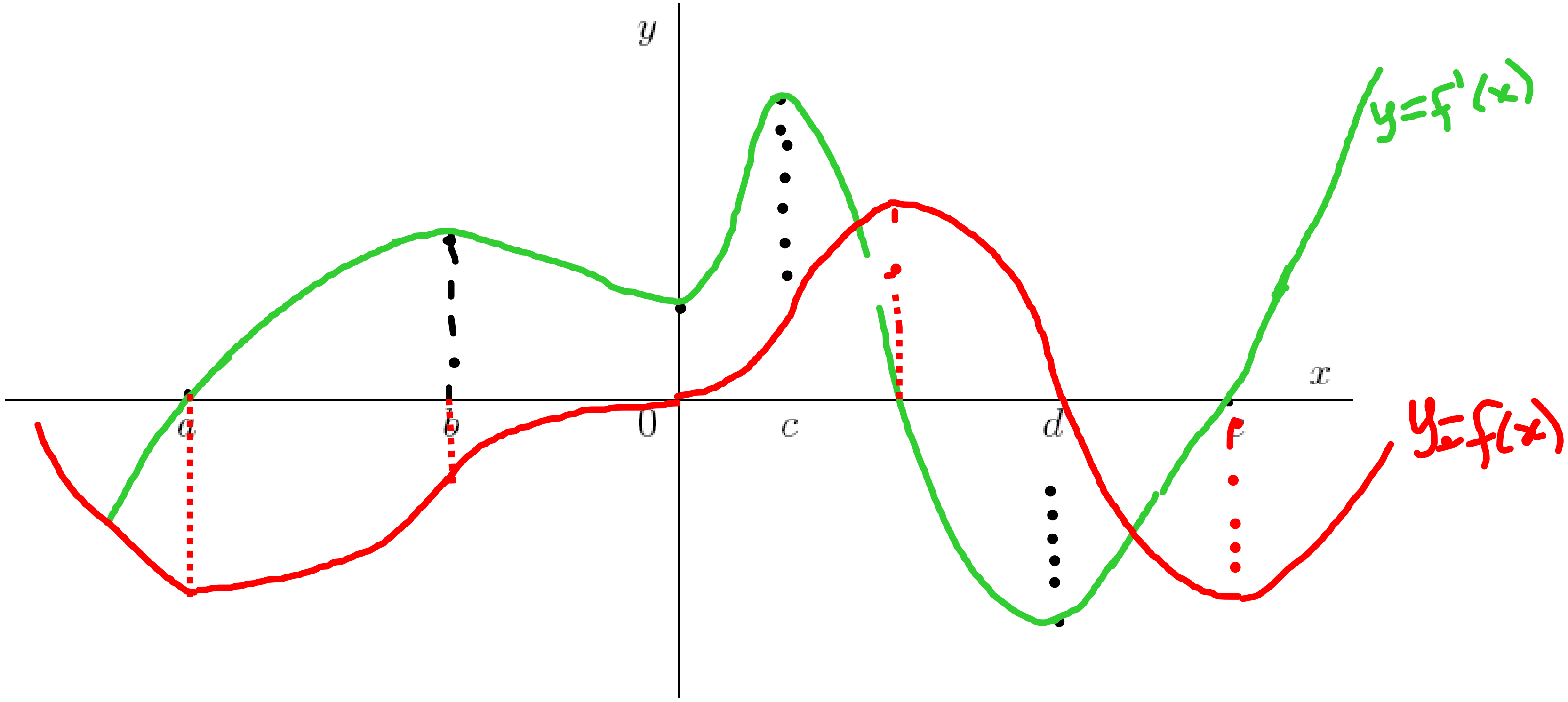


f' is decreasing on $(-\infty, -2) \cup (0, +\infty)$. Hence, $f'' < 0$ on those intervals and therefore $f(x)$ is concave downward on $(-\infty, -2) \cup (0, +\infty)$.

f' is increasing on $(-2, 0)$. Hence $f'' > 0$ on $(-2, 0)$ and $\Rightarrow$ $f(x)$ is concave upward on $(-2, 0)$

Inflection points : $x = -2, 0$.

EXAMPLE 5. *Consider the graph of the* **derivative** $f'(x)$ *of some function* f.



Given that $f(0) = 0$, *sketch a possible graph of* f.