26: Homogeneous Linear Systems with Constant Coefficients (sec. 7.5)

1. **Proposition.** *If λ is an eigenvalue of matrix A and v is an eigenvector corresponding to this eigenvalue then*

$$X(t) = e^{\lambda t} v$$

is a solution of the system $X' = AX$, i.e solution of the homogeneous linear system with constant coefficients.

Real Distinct Eigenvalues

2. *FACT: If A has n <u>distinct real</u> eigenvalues $\lambda_1, \ldots, \lambda_n$ and $v_1, \ldots, v_n$ are the corresponding eigenvectors, then*

$$\{e^{\lambda_1 t} v_1, \ldots, e^{\lambda_n t} v_n\}$$

is a fundamental set of solutions and the general solution is

$$X(t) = C_1 e^{\lambda_1 t} v_1 + \ldots + C_n e^{\lambda_n t} v_n.$$

5. EXAMPLE. Find general solution of the system

$$\begin{aligned} x_1' &= x_1 - x_2 + 4x_3 \\ x_2' &= 3x_1 + 2x_2 - x_3 \\ x_3' &= 2x_1 + x_2 - x_3 \end{aligned}$$

By Example 7 (Notes #25) we know that the coefficient matrix A has 3 real distinct eigenvalues:

$$\lambda_1 = 1, \ \lambda_2 = -2, \ \lambda_3 = 3$$

The corresponding eigenvectors are

$$v^1 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}, \quad v^2 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \quad v^3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

General solution

$$X(t) = c_1 e^{t} \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + c_3 e^{3t} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$