MATH 171 FALL 2014 EXAM 2 PARTIAL SOLUTIONS

1. (a) $\frac{dy}{dx} = 2x\sec^2(x^2)$

(b) $\frac{dy}{dx} = x^3\cos x + 3x^2\sin x$

(c) $\frac{dy}{dx} = \frac{e^{2x} - (x+1)(2e^{2x})}{e^{4x}}$

$= \frac{e^{2x}(1-2x-2)}{e^{4x}}$

$= \frac{-1-2x}{e^{2x}}$

(d) $\frac{dy}{dx} = \sec(\sqrt{x^2+1})\tan(\sqrt{x^2+1})\left(\frac{1}{2}(x^2+1)^{-1/2}\right)(2x)$

$= \frac{x}{\sqrt{x^2+1}}\sec(\sqrt{x^2+1})\tan(\sqrt{x^2+1})$

2. $3y^2\frac{dy}{dx} + x\left(2y\frac{dy}{dx}\right) + y^2 - 2x = 0$

$\frac{dy}{dx}(3y^2 + 2xy) = 2x - y^2$ $\qquad \frac{dy}{dx} = \frac{2x-y^2}{3y^2+2xy}$

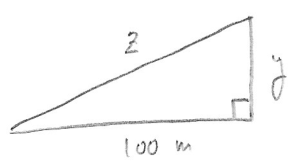
3. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-\sin t}{1}$, set $1-\sin t = 0$ to get $\sin t = 1$

$t = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

so $x = 2k\pi$, $y = \frac{\pi}{2} + 2k\pi + \cos\left(\frac{\pi}{2} + 2k\pi\right)$

$= \frac{\pi}{2} + 2k\pi$

The points are $\left(2k\pi, \frac{\pi}{2} + 2k\pi\right)$, $k \in \mathbb{Z}$.

4. 


$\frac{dy}{dt} = 5$ m/s

Find $\frac{dz}{dt}$ when $y = 50$ m.

When $y = 50$ m:
$100^2 + 50^2 = z^2$
$z = 50\sqrt{5}$

$100^2 + y^2 = z^2$

$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2 \cdot 50 \cdot 5 = 2 \cdot 50\sqrt{5} \cdot \frac{dz}{dt}$ so $\frac{dz}{dt} = \sqrt{5}$ m/s $\approx 2.236$ m/s

5. (a) By a sum of angles identity, the limit is $\lim_{x\to 0} \frac{\cos x\cos\frac{\pi}{2} - \sin x\sin\frac{\pi}{2}}{x}$

$= \lim_{x\to 0}\left(-\frac{\sin x}{x}\right) = -1$ (since $\lim_{x\to 0}\frac{\sin x}{x} = 1$)

(b) By a property of logarithms, the limit is $\lim_{x\to 2}\left(\log_2\left(\frac{x^2-4}{x-2}\right)\right) = \lim_{x\to 2}\log_2(x+2)$

$= \log_2(2+2)$

$= 2$