MATH 171H FALL 2014 EXAM 2 PARTIAL SOLUTIONS

1. (a) $\frac{dy}{dx} = -x^2 \sin x + 2x\cos x$

(b) $\frac{dy}{dx} = \frac{e^{2x} - (x+1)(2e^{2x})}{e^{4x}} = \frac{e^{2x}(1-2x-2)}{e^{4x}} = \frac{-1-2x}{e^{2x}}$

(c) $\frac{dy}{dx} = \sec(\sqrt{x^2+1})\tan(\sqrt{x^2+1})\left(\frac{1}{2}(x^2+1)^{-1/2}\right)(2x)$

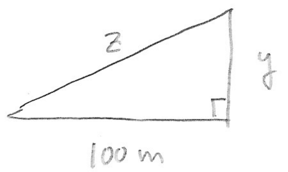
$= \frac{x}{\sqrt{x^2+1}}\sec(\sqrt{x^2+1})\tan(\sqrt{x^2+1})$

2. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-\sin t}{1}$, set $1-\sin t = 0$ to get $\sin t = 1$, so $t = \frac{\pi}{2}+2k\pi$, $k \in \mathbb{Z}$.

Then $x = 2k\pi$, $y = \frac{\pi}{2} + 2k\pi + \cos\left(\frac{\pi}{2}+2k\pi\right) = \frac{\pi}{2}+2k\pi$.

The points are $\left(2k\pi, \frac{\pi}{2}+2k\pi\right)$, $k \in \mathbb{Z}$.

3.



$\frac{dy}{dt} = 5$ m/s

Find $\frac{dz}{dt}$ when $y = 50$ m.

When $y = 50$ m:

$100^2 + 50^2 = z^2$

$z = 50\sqrt{5}$

$100^2 + y^2 = z^2$

$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2 \cdot 50 \cdot 5 = 2 \cdot 50\sqrt{5}\,\frac{dz}{dt}$ so $\frac{dz}{dt} = \sqrt{5}$ m/s ≈ 2.236 m/s

4. (a) By a sum of angles identity, the limit is $\lim_{x\to 0} \frac{\cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}}{x}$

$= \lim_{x\to 0}\left(-\frac{\sin x}{x}\right) = -1$ (since $\lim_{x\to 0}\frac{\sin x}{x} = 1$)

(b) By a property of logarithms, the limit is $\lim_{x\to 2} \log_2\left(\frac{x^2-4}{x-2}\right) = \lim_{x\to 2}\log_2(x+2)$

$= \log_2(2+2) = 2$

5. (a) $f(x) = g(x) + h(x)$ where $g(x) = x^{50}$ and $h(x) = \sin x$.

Since g is a polynomial of degree 50, $g^{(100)}(x) = 0$ (the degree drops by 1 each time a derivative is taken, until we obtain a constant function, whose derivative is 0).

Now $h'(x) = \cos x$, $h''(x) = -\sin x$, $h'''(x) = -\cos x$, $h''''(x) = \sin x$, so the higher derivatives repeat. Since 100 is divisible by 4, $h^{(100)}(x) = \sin x$. Therefore $f^{(100)}(x) = \sin x$.

(b) $f'(x) = 2e^{2x}$, $f''(x) = 4e^{2x}$, etc. Each time a derivative is taken, another factor of 2 appears. So $f^{(100)}(x) = 2^{100}e^{2x}$.

6. (a) $f(x) = \cos|x| = \cos x$ (since $\cos(-x) = \cos x$ for all x).

Therefore f is differentiable at 0.

(b) No. If it were, then $f'(0) = \lim_{h\to 0}\frac{\sin|h| - \sin 0}{h} = \lim_{h\to 0}\frac{\sin|h|}{h}$, but

$\lim_{h\to 0^+}\frac{\sin|h|}{h} = \lim_{h\to 0^+}\frac{\sin h}{h} = 1$, while

$\lim_{h\to 0^-}\frac{\sin|h|}{h} = \lim_{h\to 0^-}\left(\frac{\sin(-h)}{h}\right) = \lim_{h\to 0^-}\left(-\frac{\sin h}{h}\right) = -1.$

Therefore the (two-sided) limit does not exist.

One can see from the graph of $f(x) = \sin|x|$ that there is a cusp at $x = 0$.