Math 366 Lecture Notes
Section 11.2 – Polygons

Draw a path on a piece of paper without lifting the pencil and without retracing any part of the path except single points. The drawing is called a curve. (see p. 698).

A simple curve is one that does not intersect itself.

A closed curve is one with no endpoints and which completely encloses an area.

A convex curve is a simple, closed curve in which any straight line that crosses the curve crosses it at just two points.

A concave curve is one that is not convex.

A polygon is a simple, closed curve made by joining line segments, where each line segment intersects exactly two others. Notation: \( ABCD, BCDA \), etc. (where \( A, B, C, \) and \( D \) are consecutive vertices)

An \( n \)-gon is a polygon with \( n \) sides.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
</tr>
<tr>
<td>7</td>
<td>Heptagon</td>
</tr>
<tr>
<td>8</td>
<td>Octagon</td>
</tr>
<tr>
<td>9</td>
<td>Nonagon</td>
</tr>
<tr>
<td>10</td>
<td>Decagon</td>
</tr>
</tbody>
</table>

Every simple closed curve separates the plane into three disjoint subsets: the interior of the curve, the exterior of the curve, and the curve itself.
A polygon and its interior make up a **polygonal region**.

Look at the figure below (Billstein, 9th ed, p. 591). Determine whether point X is inside or outside the simple closed curve. Explain your reasoning so that it can be generalized to other simple closed curves.

Any two sides of a polygon having a common vertex determine an **interior angle** or **angle of the polygon**. An **exterior angle of a convex polygon** is determined by a side of the polygon and the extension of a contiguous side of the polygon.

A **diagonal** is a line segment connecting nonconsecutive vertices of a polygon.

**Congruent Segments and Angles**

**Congruent** parts are parts with the same size and shape.

**Congruent segments** can be fitted exactly one top of each other. (They have the same length.) Notation: $\overline{AB} \cong \overline{CD}$ ($\overline{AB} \cong \overline{CD}$ iff $AB = CD$)

**Congruent angles** have the same measure. ($\angle A \cong \angle B$ iff $m \angle A = m \angle B$)

**Note:** “iff” means “if and only if”
**Regular Polygons**

A *regular polygon* is one in which all the interior angles are congruent and all the sides are congruent. A regular polygon is both equiangular and equilateral. (see p. 701)

**Triangles and Quadrilaterals** (see p. 702)

A triangle containing one right angle is a **right triangle**.

A triangle in which all the angles are acute is an **acute triangle**.

A triangle containing one obtuse angle is an **obtuse triangle**.

A triangle with no congruent sides is a **scalene triangle**.

A triangle with at least two congruent sides is an **isosceles triangle**.

A triangle with three congruent sides is an **equilateral triangle**.

A **trapezoid** is a quadrilateral with at least one pair of parallel sides. (Some texts define a trapezoid as a quadrilateral with exactly one pair of parallel sides.)

A **kite** is a quadrilateral with two adjacent sides congruent and the other two sides also congruent.

An **isosceles trapezoid** is a trapezoid with exactly one pair of congruent sides. (Equivalently, an isosceles trapezoid is a trapezoid with two congruent base angles.)

A **parallelogram** is a quadrilateral in which each pair of opposite sides is parallel.

A **rectangle** is a parallelogram with a right angle. (Equivalents, a rectangle is a quadrilateral with four right angles.)

A **rhombus** is a parallelogram with two adjacent sides congruent. (Equivalently, a rhombus is a quadrilateral with all sides congruent.)

A **square** is a rectangle with two adjacent sides congruent. (Equivalently, a square is a quadrilateral with four right angles and four congruent sides.)

Why might your textbook define some of the quadrilaterals differently from student textbooks, such as the one on p. 704?
Hierarchy Among Polygons

Relate the polygons by hierarchy, using a Venn Diagram or a tree.

Which of the following are true?

- An equilateral triangle is isosceles.
- A square is a regular quadrilateral.
- If one angle of a rhombus is a right angle, then all the angles of the rhombus are right angles.
- A square is a rhombus with a right angle.
- All the angles of a rectangle are right angles.
- A rectangle is an isosceles trapezoid.
- If a kite has a right angle, then it must be a square.