Math 366 Lecture Notes
Section 14.1 – Translations and Rotations

Translations

Any rigid motion that preserves length or distance is an isometry (meaning “equal measure”).

Any function from a plane to itself that is a one-to-one correspondence between a plane and itself is a transformation of the plane.

A translation is a motion of a plane that moves every point of the plane a specified distance in a specified direction along a straight line.

Properties of Translations

• A figure and its image are congruent.

• The image of a line is a line parallel to it.

Constructions of Translations

Construct the image $A'$ of point $A$ in the direction and magnitude of vector $\overrightarrow{MN}$.

1) Draw ray $MA$.

2) Construct ray $AP$ parallel to $\overrightarrow{MN}$.

3) Construct point $A'$ on $AP$ the same distance from $A$ as $M$ is from $N$. 
A geoboard or a grid can be used to find an image of a set of points.

Find the image of $\overline{AB}$ under the translation from $X$ to $X'$ in the figure below.

[Diagram showing a grid with points A, B, and X']

**Coordinate Representation of Translations**

In the figure below, $\triangle A'B'C'$ is the image of $\triangle ABC$ under the translation defined by the slide arrow from $O$ to $O'$, where $O$ is the origin and $O'$ has coordinates $(5, -2)$.

[Diagram showing $\triangle ABC$ and $\triangle A'B'C'$]

The table below shows how the coordinates of the image vertices are obtained from the original vertices.

<table>
<thead>
<tr>
<th>Point $(x, y)$</th>
<th>Image Point $(x + 5, y - 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A (-2, 6)$</td>
<td>$A' (3, 4)$</td>
</tr>
<tr>
<td>$B (0, 8)$</td>
<td>$B' (5, 6)$</td>
</tr>
<tr>
<td>$C (2, 3)$</td>
<td>$C' (7, 1)$</td>
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</tbody>
</table>
Theorem 14-1: Translation in a Coordinate System
A translation is a function from the plane to the plane such that to every point \((x, y)\) corresponds the point \((x + a, y + b)\), where \(a\) and \(b\) are real numbers.
Notation: \((x, y) \rightarrow (x + a, y + b)\)

Find the coordinates of the image of the vertices of quadrilateral \(ABCD\) in the figure below under the translations given. Draw the image in each case.

- \((x, y) \rightarrow (x + 2, y - 3)\)
- a translation determined by the slide arrow from \(A(1, 2)\) to \(A'(3, -1)\)

A rotation, or turn, is another kind of isometry. The figure below illustrates congruent figures that resulted from a rotation about point \(O\). Point \(O\) is the turn center, and \(\angle O\) is the turn angle.

A rotation can be constructed with tracing paper.
A rotation is a transformation of the plane determined by holding one point, the center, fixed and rotating the plane about this point by a certain amount in a certain direction. Usually a positive rotation is counterclockwise, and a negative rotation is clockwise.

A rotation of 360° about a point will move any point (and figure) onto itself. Such a transformation is an identity transformation. A rotation of 180° is a half-turn. Rotations are useful in determining turn symmetries.

Construction of a Rotation

Construct the image of point $P$ under a rotation with center $O$ through the angle and in the direction given in the figure below.

1) Construct an isosceles triangle $BAC$ with $B$ on one side of the given angle and $C$ on the other side so that $AB = AC = OP$.
2) Construct $\triangle POP'$ congruent to $\triangle BAC$.

Slopes of Perpendicular Lines

Theorem 14-2: Slopes of Perpendicular Lines

Two lines, neither of which is vertical, are perpendicular iff their slopes $m_1$ and $m_2$ satisfy the condition $m_1m_2 = -1$. Every vertical line has an undefined slope and is perpendicular to a line with slope 0.

Are the lines $2x - 3y = 7$ and $3x - 2y = 5$ parallel, perpendicular, or neither?

Find the equation of the line through (-3, 2) and perpendicular to the line $3x + y = 4$.  