Math 151H
Sections 201 and 202
Final Exam

13 December 2006
Instructor: F. Sottile

Full credit is given only for complete and correct answers.
No aids allowed on the exam. Please write your answers in blue books.
Do persevere; partial credit will be given, and you are all good students.
Point totals are in brackets next to each problem. 200 points total

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1. [15] Suppose that f is a function and l, a are real numbers. Give the precise ϵ-δ definition of *limit*. That is, give the definition of: "The function f approaches the limit l near a".

2. [15] Use the definition of the limit and give an ϵ–δ proof that

$$\lim_{x\to 4} \sqrt{x} \;=\; 2\,.$$

3. [15] Recall that a function f is *differentiable* at a point a if the limit

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

exists, and in that case we write $f'(a)$ for the value of this limit.

Use this and the limit laws (no ϵ–δ!) to compute $f'(2)$, where $f(x)=\frac{1}{x}$.

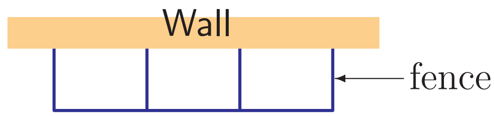
4. [10] Give a vector (parametric) equation $\mathbf{r}(t)$ for the line passing through the point $\mathbf{r}_0 \in \mathbb{R}^2$ having direction $\mathbf{v}$.

5. [15] The position $\mathbf{r}$ of the sun with respect to the Earth is

$$\mathbf{r}(t) \;=\; \langle A\cos(\tfrac{2\pi}{365}t), A\sin(\tfrac{2\pi}{365}t)\rangle\,.$$

(A is the Astronomical unit.) In the Heraclidean model of the solar system (where the Sun revolves around the Earth) what is the acceleration of the Sun?

6. [20] To pen his pet pigs, Frank is building backyard enclosures for them in the shape shown below (taking advantage of an exsiting wall).



If he has 40 metres of fence, what is the largest area he can enclose?

7. [45] Compute the derivatives with respect to the variable x of the following functions.

   (a) $\log(x)$

   (b) e^{x^2}

   (c) $\sin(e^x)$

   (d) $e^{\sin(x)}$

   (e) $\sqrt{\sin(x^2) + \sin\sqrt{x}}$

   (f) $\sin(e^x + x^2\cos(x + e^x))$

   (h) $f(x) := \int_1^x \frac{dt}{t}$

   (i) $f(x) := \int_1^{x+\sin(x)} \frac{dt}{t}$

8. [15] Set $f(x) := xe^{-x^2}$ for $x \in [-2, 2]$. Determine the maximum and minimum values of f and state where it is increasing and decreasing.

9. [15] State Lagrange's Mean Value Theorem. Include the hypotheses and the conclusion.

10. [20] The Sun is setting behind a 120-metre tall building. How fast (in metres per minute) is the shadow lengthening when the Sun makes an angle of $\pi/6$ with the horizon (i.e. 2 hours before sunset)?

11. [15] What is wrong with the following use of L'Hôpital's Rule?

$$\lim_{\omega\to 1} \frac{\omega^3 + \omega - 2}{\omega^2 - 3\omega + 2} = \lim_{\omega\to 1} \frac{3\omega^2 + 1}{2\omega - 3} = \lim_{\omega\to 1} \frac{6\omega}{2} = 3\,.$$