Math 151H
Sections 201 and 202
Second Test

26 October 2006

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Full credit is given only for complete and correct answers.
No aids allowed on the exam. Please write your answers in blue books.
Do persevere; partial credit will be given, and you are all good students.
Point totals are in brackets next to each problem.

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1. [10] Suppose that f is a function defined on a neighborhood of a point a. What does it mean for f to be differentiable at a? What is its derivative at a?

2. [15] Use the definition of derivative to compute $f'(a)$, where $f(x) = \frac{1}{x}$ and $a \neq 0$.

3. [10] Find the equation of the tangent line to the curve $y^2 = x^3(2 - x)$ at the point $(1, 1)$.
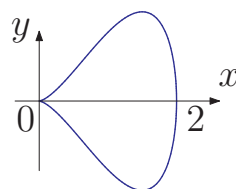
4. [10] Compute the trigonometric limit, $\lim\limits_{\theta \to 0} \frac{1 - \cos\theta}{\theta \tan\theta}$.

5. [15] Absent-mindedly leaning against the chalkboard, Frank's feet begin to slip at a rate of 10 inches per second. If he is 70 inches tall, how fast is his head moving downwards when his feet are 40 inches from the wall? At what speed does his head hit the floor?

70 inches

$\dot{x} = 10$ inches/sec
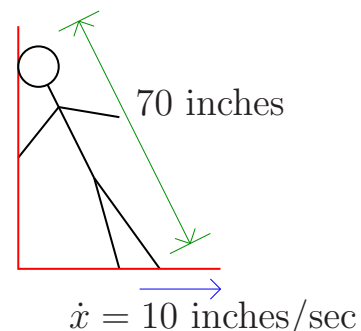
6. [10] Let f be a function and $a \in \mathbb{R}$. Indicate whether each of the following statements is true or false. You do **not** need to justify your answers.

   (a) If f is continuous at a point a, then it is differentiable at a.

   (b) If f is differentiable at a point a, then it is continuous at a.

   (c) If f is differentiable on an interval, then f' is continuous on that interval.

7. [30] Compute derivatives with respect to the variable x of the following functions.

   (a) $\sqrt{x + \sqrt{x + \sqrt{x}}}.$

   (b) $\sin\big((x + 1)^2(x + 2)\big).$

   (c) $\dfrac{\cos(1 + x)}{\sin(1 - x)}.$

   (d) $\left(x - \frac{1}{x}\right)^{3/2}.$

   (e) $\sin(x \cos(x)).$

   (f) $t^\pi - \sin(e).$