**d'Alembert's solution of the wave equation**

1. Introduce new variables (coordinates) by

$$u = s - x, \quad v = s + x.$$

Show that the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial s^2}$$

is equivalent to

$$\frac{\partial^2 y}{\partial u \, \partial v} = 0. \tag{1}$$

*Hint:* By the multivariable chain rule, for any function $f$,

$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x}\frac{\partial f}{\partial u} + \frac{\partial v}{\partial x}\frac{\partial f}{\partial v}\,.$$

2. Show that the solutions of (1) are the functions of the form

$$y = \Psi(u) + \Phi(v).$$

*Hint:* When is a "constant" of integration not a constant?

3. Substitute the definitions of $u$ and $v$ and declare victory.