Foundations of Mathematics — Math 300 Sections 902, 905

Tuesday 22 September 2020 — Class worksheet

Definition. An integer a is *even* if there is an integer k such that $n = 2k$. An integer a is *odd* if there is an integer k such that $n = 2k+1$. (We assume the following result, which we cannot yet prove: Every integer n is either even or odd.

Definition. Let a be a nonnegative real number. The *square root of a*, written $\sqrt{a}$ is the unique positive real number r such that $r^2 = a$.
(It is a theorem in analysis that every nonnegative real number has a nonnegative square root, and we are assuming this for this definition.)

1. Prove the following by contradiction *reductio ad absurdum*:
   For all integers n, if n^2 is odd, then n is odd.
2. Prove the following by contradiction *reductio ad absurdum*:
   For all real numbers a and b with $b \geq 0$, if $a^2 \geq b$, then either $a \geq \sqrt{b}$ or $a \leq -\sqrt{b}$.
3. Using the definitions, prove by cases that for every integer n, $n^2 - n + 41$ is odd.