Foundations of Mathematics — Math 300 Sections 902, 905

Tuesday 29 September 2020 — Concept Quiz

Answers to Concept Quiz 4.1.2

1. **Principle of Mathematical Inductionubset version .**

   Suppose that T is a subset of $\mathbb{N}$. The *Principle of Mathematical Induction* allows us to conclude/assert that $T = \mathbb{N}$ if we can show that T has two properties. What are these properties?

   Property/Step 1:

   $1 \in T$.

   Property/Step 2:

   For every positive integer k, if $k \in T$, then $k + 1 \in T$.

2. **Summation Notation.**

   Recall the notation for summation (you saw this in Calculus). Suppose that $f(i)$ is a function of an integer i. Let b be an integer. Then $\sum_{a=1}^{b} f(a)$ is shorthand for $f(1) + \cdots + f(b)$. That is, we substitute each integer between 1 and b for the variable a in the function/expression $f(a)$ and add all of these up. The following will test your understanding of this notation. All symbols, $i, j, k, \ell, n, \ldots$ are positive integers.

   (a) What is $\sum_{i=2}^{5} i$ equal to?

   This is $2 + 3 + 4 + 5 = 14$.

   (b) What is $\sum_{n=0}^{k} 2^n$ equal to?

   This is $2^0 + 2^1 + \cdots + 2^k = 1 + 2 + \cdots + 2^k = 2^{k+1} - 1$.

   (c) What is $\sum_{j=1}^{n} \ell^2$ equal to?

   This is $\ell^2 + \ell^2 + \cdots + \ell^2$ (n summands), so it is $n\ell^2$.