History of Mathematics — Math 629

Third Homework: 30 January 2024

Due Monday 5 February 2024.

Some of these problems will require you to find other material than just what is in the readings. To hand in: We are using Gradescope for homework submission.

1. Stillwell gives Euclid's elegant proof that there are infinitely many prime numbers, one of the great proofs in mathematics. Find and describe/explain a *different* proof that there are infinitely many prime numbers.

2. Continued fractions. Do the exercises in Stillwell: 3.4.1, 3.4.2, 3.4.3, and 3.4.4.

3. Let $y = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$. What is y?

4. Consider Archimedes' quote from The Method: "It is of course easier to supply the proof when we have previously acquired some knowledge of the questions by the method, than it is to find it without any previous knowledge." What was "the method" he is referring to? What does his quote say about the role of experimentation or studying examples in Mathematics?

5. Exhaustion. Do the exercises in Stillwell's Section 4.4 (4.4.1, 4.4.2, and 4.4.3) to prove the formula for the logarithm of product of rational numbers. Make sure to use exhaustion and not calculus tricks. (Why did I restrict this to rational numbers a and b?)

   Hint: From the definition of exhaustion, to show two quantities are equal, say $\log(a)$ and $\log(ab) - \log(b)$, is to show how any lower approximation to one can be transformed into a lower approximation for the other, with the same area, and the same for upper approximations; I find the method here to be quite elegant. (Note that, as a mathematician, I use log for the logarithm with respect to the natural base, Euler's number e.)