# Excitation of an Atom by a Uniformly Accelerated Mirror through Virtual Transitions

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We find that uniformly accelerated motion of a mirror yields excitation of a static two-level atom with simultaneous emission of a real photon. This occurs because of virtual transitions with probability governed by the Planck factor involving the photon frequency $\nu$ and the Unruh temperature. The result is different from the Unruh radiation of an accelerated atom, which is governed by the frequency of the atom, $\omega$, rather than frequency of the emitted photon. We also find that the excitation probability oscillates as a function of the atomic position because of interference between contributions from the waves incident on and reflected from the mirror.

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*Introduction.*—Virtual processes are part of the vacuum picture of quantum electrodynamics. E.g., an atom can jump to an excited state and a virtual photon is emitted, followed quickly by the reverse process, in which the atom jumps back to the ground state and now absorbs a photon. The surreal virtual processes have real effects; e.g., they can shift the energy levels of atoms (Lamb shifts) and yield van der Waals forces. Virtual processes contribute to Raman scattering, which is of great practical importance for spectroscopy. Namely, in one of the pathways, the molecule can go into a virtual state and at the same time emit a Raman photon, and then a higher-frequency pump photon is absorbed. The excitation of the molecule and emission of the photon take place before absorption and are due to counterrotating terms in the Hamiltonian.

Spontaneous creation of particles in an external field or a curved spacetime is one of the most prominent phenomena in quantum field theory. A strong electric field produces pairs of charged particles and antiparticles, known as the Schwinger mechanism [1]. Another remarkable phenomenon is the emission of all species of particles from the strongly curved spacetime of hypothetical black holes, known as Hawking radiation [2].

For a free quantum field in its vacuum state in Minkowski spacetime, an observer with uniform acceleration $a$ will feel that he is bathed by a thermal distribution of quanta of the field (Rindler particles) at temperature [3]

$$T_U = \frac{\hbar a}{2\pi k_B c}. \tag{1}$$

In particular, ground-state atoms, accelerated through the Minkowski vacuum, will be promoted to an excited state by absorption of the Rindler particles (Unruh effect) [3]. The inertial observer interprets the absorption of a Rindler particle as the emission of a Minkowski particle [4], which is known as acceleration radiation.

By breaking and interrupting the virtual processes which take place all around us, we can render the virtual photons real. Atom acceleration converts virtual photons into real ones at the expense of the energy supplied by the external force field driving the center-of-mass motion of the atom against radiation reaction. One can enhance the chance of photon emission by many orders of magnitude by turning on coupling between the field and atom very quickly [5]. This can be achieved when atoms are injected into a high-$Q$ cavity which produces a strong nonadiabatic effect at the cavity boundaries [5]. When the atoms are injected in a regular fashion, squeezed radiation can be produced [5].

Spontaneous excitation of atoms in the curved spacetime of Schwarzschild [6–8] and Kerr [9] black holes, and van der Waals or Casimir-Polder interatomic interactions between two accelerating atoms [10–12] are subjects of recent interest.

The quantum vacuum can also be excited by moving mirrors [13]. If the mirrors move over a limited time interval, the "in" vacuum state generally contains photons afterwards, and the "out" vacuum state contained photons previously. This is now known as the dynamical Casimir effect. The number of generated photons is determined by how fast the mirror velocity changes with time. For adiabatic motion the effect is tiny unless the mirrors move near the speed of light. Nevertheless, the dynamical Casimir effect has been demonstrated experimentally in a superconducting circuit [14].

A single mirror oscillating at frequency $f$ produces photons in pairs such that $f_1 + f_2 = f$. This is similar to an optical parametric oscillator that converts an input laser wave into two output waves of lower frequency by means of the second-order nonlinear optical interaction. It has been demonstrated experimentally that an oscillating mirror generates squeezed light, which is a signature of the quantum nature of the generation process [14].

Theoretical investigations of photon generation by accelerating mirrors have involved generalization of the problem into $3+1$ dimensions [15–17], study of the backreaction on the mirrors [18,19], nonplanar mirror shapes [15,16,20], "mirrors" carrying different boundary conditions [21,22], and analogy with radiation from an electrical charge in classical electrodynamics [15,17].

Here we consider a system that consists of a mirror and an atom, and we investigate excitation of such a system by virtual processes when there is relative acceleration between the atom and the mirror. Namely, we compare two cases. First is the radiation of an accelerating atom in the presence of a fixed mirror when modes of the field are stationary. In the second case the atom is fixed, but the field modes are changing with time (mirror is accelerating). As we show, excitation occurs with a different probability in either case, but the answers are related by interchange of the atom and photon frequencies. This provides new insights on how the equivalence principle applies to QED. In addition, we find that interference between incident and reflected virtual photons yields spatial oscillations of the excitation probability.

*Excitation of an atom uniformly accelerated relative to a mirror.*—Here we consider an electrically neutral two-level ($a$ and $b$) atom with transition angular frequency $\omega$ moving along the $z$ axis with a uniform acceleration $a$. The atom trajectory is given by

$$t(\tau)=\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right),\quad z(\tau)=\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right), \tag{2}$$

where $t$ is the lab time and $\tau$ is the proper time for the accelerated atom. In this Letter we consider either dimension $1+1$ or dimension $3+1$, but we restrict photons to have wave vector $\mathbf{k}$ parallel to the $z$ axis. The interaction Hamiltonian between the atom and a photon with angular frequency $\nu$ reads

$$\hat{V}(\tau)=\hbar g(\hat{a}_\nu\phi_\nu[t(\tau),z(\tau)]+\mathrm{H.c.})(\hat{\sigma}e^{-i\omega\tau}+\mathrm{H.c.}), \tag{3}$$

where $\hat{a}_\nu$ is the photon anihilation operator, $\hat{\sigma}$ is the atomic lowering operator, and $g$ is the atom-field coupling constant. We shall assume that $g$ is approximately independent of $\tau$, which is the case for scalar (spin-0) "photons" (however, $g$ may depend on $\nu$). Initially the atom is in the ground state, and there are no photons. In Eq. (3), $\phi_\nu$ is the field mode function. Since the atom feels the local value of the field, $\phi_\nu$ is taken at the atom's location $t(\tau)$, $z(\tau)$.

We assume that there is a plane mirror fixed at $z=z_0<c^2/a$, so that the atom's trajectory does not cross the mirror (see Fig. 1). Normal modes of the electromagnetic field with frequency $\nu$ are standing waves

$$\phi_\nu=e^{-i\nu t-ikz+ikz_0}-e^{-i\nu t+ikz-ikz_0},\quad \nu,k>0, \tag{4}$$

which obey the boundary condition $\phi_\nu(z_0)=0$. They are superpositions of incident and reflected waves. We assume

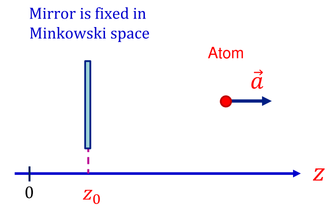


FIG. 1. The atom is moving with uniform acceleration $a$ along the $z$ axis, and the mirror is fixed at $z=z_0$ in Minkowski spacetime.

that the normalization factor in $\phi_\nu$ is subsumed under the coupling constant $g$.

The probability of excitation of the atom (angular frequency $\omega$) with the simultaneous emission of a photon with angular frequency $\nu$ is due to a counterrotating term $\hat{a}_\nu^+\hat{\sigma}^+$ in the interaction Hamiltonian. The probability of this event is

$$P_{\mathrm{exc}}=\frac{1}{\hbar^2}\left|\int d\tau\langle 1_\nu,a|\hat{V}(\tau)|0,b\rangle\right|^2$$
$$=g^2\left|\int_{-\infty}^{\infty}d\tau[e^{ikz(\tau)-ikz_0}-\mathrm{c.c.}]e^{i\nu t(\tau)+i\omega\tau}\right|^2. \tag{5}$$

Inserting here Eq. (2) and $k=\nu/c$, we obtain

$$P=g^2\left|\int_{-\infty}^{\infty}d\tau[e^{i\nu(c/a)e^{a\tau/c}-ikz_0}e^{i\omega\tau}-\mathrm{c.c.}]\right|^2.$$

Making a change of the variable to $x=[(\nu c)/a]e^{a\tau/c}$ yields

$$P=\frac{c^2g^2}{a^2}\left|\int_0^{\infty}dx[e^{ix}x^{[(ic\omega)/a]-1}e^{-ikz_0+i[(c\omega)/a]\ln[a/(\nu c)]}-\mathrm{c.c.}]\right|^2. \tag{6}$$

Taking into account that

$$\int_0^{\infty}dxe^{-ix}x^{-[(ic\omega)/a]-1}=e^{-[(\pi c\omega)/2a]}\Gamma\left(-\frac{ic\omega}{a}\right),$$

where $\Gamma$ is the gamma function, and the property $|\Gamma(-ix)|^2=\pi/[x\sinh(\pi x)]$, we obtain

$$P=\frac{8\pi cg^2}{a\omega}\frac{\sin^2(\nu z_0/c+\varphi)}{\exp(\frac{2\pi\omega c}{a})-1}, \tag{7}$$

where $\varphi$ is independent of $z_0$.

We see that $P$ is proportional to the Planck factor $\{\exp[(\hbar\omega)/(k_BT_U)]-1\}^{-1}$, which contains the frequency of the atom $\omega$ and the Unruh temperature [Eq. (1)]. $P$ oscillates as a function of the mirror position $z_0$ because of interference between contributions from the incident and reflected waves. This is somewhat analogous to Fano interference [23]. The period of the spatial oscillations is equal to $\lambda/2$, where $\lambda$ is the wavelength of the emitted photon.

*Excitation of the atom by a uniformly accelerated mirror.*—Next, we consider the opposite case—namely, we assume the atom does not move in the inertial reference frame. It is fixed at $z = z_0 < c^2/a$, and the mirror is uniformly accelerated following the trajectory in Eq. (2) [see Figs. 2(a) and 2(b)]. The coordinate transformation

$$t = \frac{c}{a} e^{a\bar{z}/c^2} \sinh\left(\frac{a\bar{t}}{c}\right), \tag{8}$$

$$z = \frac{c^2}{a} e^{a\bar{z}/c^2} \cosh\left(\frac{a\bar{t}}{c}\right), \tag{9}$$

where $a$ is a constant, converts the Minkowski spacetime line element $ds^2 = c^2 dt^2 - dz^2$ to the Rindler line element [24]

$$ds^2 = e^{2a\bar{z}/c^2}(c^2 d\bar{t}^2 - d\bar{z}^2). \tag{10}$$

A mirror moving along the trajectory $\bar{z} = 0$ in the Rindler space is uniformly accelerating in the Minkowski space [see Fig. 2(c)] and moves along the trajectory in Eq. (2). Normal modes of scalar photons in the conformal metric (10) take the same form as the usual positive frequency normal modes in the Minkowski metric; e.g., one can take them as standing waves

$$\phi_\nu(\bar{t}, \bar{z}) = e^{-i\nu\bar{t} + ik\bar{z}} - e^{-i\nu\bar{t} - ik\bar{z}}, \tag{11}$$

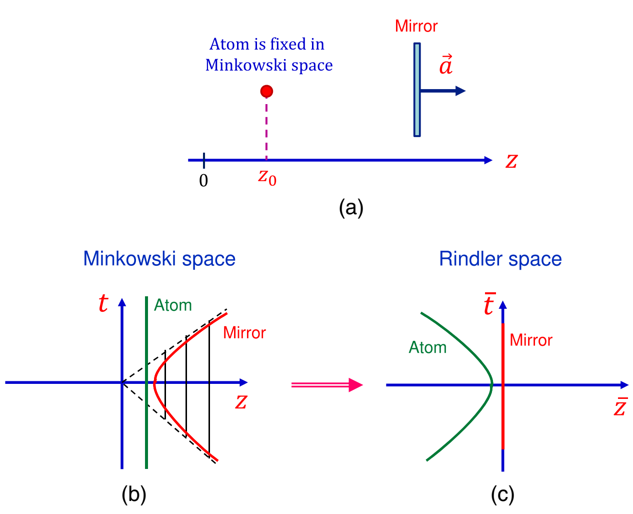


FIG. 2. (a) The mirror is moving with uniform acceleration $a$ along the $z$ axis, and the atom is fixed at $z = z_0$ in Minkowski spacetime. (b) Trajectory of the mirror and the atom in Minkowski spacetime. The mirror is moving from $z = \infty$ ($t = -\infty$) towards the atom and decelerates. At $t = 0$, the mirror reaches the turning point ($z = c^2/a$) and starts to move to the right, away from the fixed atom. For $t > 0$, the mirror is accelerating. (c) Trajectory of the mirror and the atom in Rindler space. The mirror is fixed at $\bar{z} = 0$, while the atom is moving.

where $\nu$ is the photon angular frequency in the reference frame of the mirror (Rindler space). However, the modes in Eq. (11) are a mixture of positive- and negative-frequency modes with respect to the physical Minkowski spacetime. Therefore, the vacuum state of these modes is not the Minkowski vacuum but rather the Rindler vacuum, which is what we assume for those modes.

From Eqs. (8) and (9), we obtain $\bar{t}$ and $\bar{z}$ in terms of $t$ and $z$:

$$\bar{t}(t, z) = \frac{c}{a} \operatorname{arctanh}\left(\frac{ct}{z}\right) = \frac{c}{2a} \ln\left(\frac{z + ct}{z - ct}\right), \tag{12}$$

$$\bar{z}(t, z) = \frac{c^2}{2a} \ln\left(\frac{a^2}{c^4}(z^2 - c^2 t^2)\right). \tag{13}$$

Plugging Eqs. (12) and (13) into Eq. (11) yields the mode functions in Minkowski coordinates. One should note that the coordinate transformations (12) and (13) cover only the part of the Minkowski spacetime with $z > c|t|$ (right Rindler wedge). Nevertheless, the left- and right-moving mode solutions have natural continuations into the future ($t > |z|/c$) and the past ($t < -|z|/c$) wedges respectively. Indeed, using Eqs. (12) and (13) and $k = \nu/c$, we obtain the following extension of the mode functions in Minkowski coordinates:

$$\begin{aligned}\phi_\nu(t, z) &= e^{i\nu(c/a)\ln[(a/c^2)(z - ct)]}\theta(z - ct) \\ &\quad - e^{-i\nu(c/a)\ln[(a/c^2)(z + ct)]}\theta(z + ct).\end{aligned} \tag{14}$$

Equation (14) is a superposition of the incoming (first term) and reflected (second term) traveling waves.

The probability $P$ that the static atom gets excited and a photon in the mode (14) is generated is given by the integral

$$P = g^2 \left| \int dt \phi_\nu^*(t, z_0) e^{i\omega t} \right|^2, \tag{15}$$

where $t$ is the proper time for the atom, and $z$ is taken at the atomic position $z_0$. Using Eq. (14), we obtain

$$\begin{aligned}P = g^2 \Bigg| &\int_{-\infty}^{z_0/c} dt e^{-i[(\nu c)/a]\ln[(a/c^2)(z_0 - ct)] + i\omega t} \\ &- \int_{-(z_0/c)}^{\infty} dt e^{i[(\nu c)/a]\ln[(a/c^2)(z_0 + ct)] + i\omega t} \Bigg|^2.\end{aligned} \tag{16}$$

Changing $t \to -t$ in the first term yields

$$P = g^2 \left| \int_{-(z_0/c)}^{\infty} dt e^{i[(\nu c)/a]\ln[(a/c^2)(z_0 + ct)] + i\omega t} - \text{c.c.} \right|^2. \tag{17}$$

Changing the integration variable to $x = \omega(t + z_0/c)$, we have

$$P = \frac{g^2}{\omega^2}\left|\int_0^\infty dx x^{[(i\nu c)/a]} e^{ix - i\omega z_0/c - i[(\nu c)/a]\ln[(c\omega)/a]} - \text{c.c.}\right|^2. \tag{18}$$

Using

$$\int_0^\infty dx e^{ix} x^{[(ic\nu)/a]} = -\frac{\pi e^{-[(\pi c\nu)/2a]}}{\sinh(\frac{\pi c\nu}{a})\Gamma(-\frac{ic\nu}{a})}$$

and the property $|\Gamma(-ix)|^2 = \pi/[x\sinh(\pi x)]$, we find

$$P = \frac{8\pi c\nu g^2}{a\omega^2}\frac{\sin^2(\omega z_0/c + \varphi)}{\exp(\frac{2\pi\nu c}{a}) - 1}, \tag{19}$$

where $\varphi$ is independent of $z_0$.

Equation (19) shows that the probability of atomic excitation with simultaneous emission of a photon in the mode (14) is governed by the Planck factor containing the photon frequency $\nu$. This is different from Eq. (7) obtained for the uniformly accelerated atom. In the latter case, the excitation probability is governed by the Planck factor involving the atomic frequency $\omega$. On the other hand, the spatial oscillations of the probability (19) are governed by the atomic wave number $\omega/c$, while for the case of a fixed mirror they are determined by the photon wavelength.

Our calculation in this section assumes that the mode (14) is initially empty—that is, it is in the Rindler-like vacuum associated with the mirror trajectory (see Sec. VI in Ref. [25]). A calculation for a more physically plausible initial state for the field modes would be that they are initially in the Minkowski vacuum until they reflect off the mirror. One method for calculating with such a state is given by Su *et al.* [26], but this shall be left to future work.

In any event, however, the analysis in this Letter restores a comforting symmetry between two Killing frames, and it is highly relevant to the never-ending debates about how the principle of equivalence applies to nongravitational processes in a gravitational field (see, e.g., Ref. [27]). This point is developed further in Ref. [28]. In particular, in the scenario of Ref. [8] the emptiness of the Rindler (or Boulware [29]) mode is physically natural if the experiment begins outside a massive star right before it starts to collapse.

*Summary.*—Acceleration of an atom relative to the field can lead to atomic excitation with simultaneous emission of a photon. This occurs because of virtual transitions and is governed by the counterrotating term in the interaction Hamiltonian. We found that the probability $P$ of such an event depends on whether the atom is accelerating relative to a fixed mirror or the mirror (field modes) is accelerating while the atom is held fixed. Namely, in the former (latter) case $P$ is proportional to the Planck factor containing the atom (photon) frequency and the Unruh temperature (1).

We also found that the probability $P$ undergoes spatial oscillations as a function of the atom (mirror) position due to interference between contributions from the incident and reflected waves. This is somewhat analogous to Fano interference [23]. At certain positions such interference totally suppresses photon emission along the $z$ axis.

If the system is placed in a large cavity, then the field will reach a steady state. Photon statistics can be obtained using the quantum master equation technique, as developed in the quantum theory of the laser [30]. If atoms are ejected randomly into the cavity, the photon statistics for each field mode will be thermal [5]. The average photon number in the mode, $\bar{n}_\nu$, is determined by the balance between photon emission and absorption. If the mirror is fixed and the atom is accelerating, then Eq. (7) leads to the following answer for the average photon occupation number in the mode with angular frequency $\nu$:

$$\bar{n}_\nu = \frac{1}{\exp(\frac{2\pi c\omega}{a}) - 1}. \tag{20}$$

The photon spectrum is flat; that is, $\bar{n}_\nu$ is independent of the photon frequency $\nu$. This is similar to a flat photon spectrum obtained when atoms are randomly ejected into a cavity [5].

In the opposite case of an accelerated mirror, Eq. (19) results in

$$\bar{n}_\nu = \frac{1}{\exp(\frac{2\pi c\nu}{a}) - 1}, \tag{21}$$

which is a Planck distribution. Thus, depending on whether the atom or the mirror is accelerating we obtain different photon distributions.

The result (21) is analogous to the Planck spectrum of photons emitted by atoms which are freely falling in the gravitational field of a Schwarzschild black hole [8]. In that case the covariant acceleration of atoms is equal to zero, whereas acceleration of a cavity held fixed in the Schwarzschild coordinates is nonzero. Thus, there is relative acceleration between the atoms and the field modes (cavity). This leads to the generation of acceleration radiation which to a distant observer looks much like thermal radiation with the Hawking temperature [8].

A symmetry between the excitation of a stationary atom by a mirror accelerating in Minkowski spacetime (Rindler vacuum) and an atom freely falling in the gravitational field relatively to a stationary mirror (Boulware vacuum) is a manifestation of the equivalence principle. This principle also yields a symmetry between the excitation of an atom accelerating in Minkowski spacetime relative to a stationary mirror (Minkowski vacuum) and a stationary atom excited by a mirror freely falling in a gravitational field (Hartle-Hawking vacuum).

One can test our findings experimentally in schemes that imitate an accelerating mirror [14,31] and a two-level atom [32]. E.g., one can use a superconducting transmission line microwave cavity terminated by a SQUID and coupled to an ensemble of polar molecules. The SQUID acts as an inductor whose value can be varied on very short timescales which provides the same boundary condition as the idealized moving mirror [31]. Unlike the mirror, the effective acceleration of the boundary $a$ can be much greater than $c\nu$, where $\nu$ is the frequency of microwave photons in the cavity [14].

An ensemble of $N \sim 10^4$–$10^6$ polar molecules, coherently interacting with the cavity photons, can mimic a two-level atom. The rotational excitations of molecules are in the microwave regime and have an anharmonic energy spectrum. The anharmonicity allows us to pick out a two-level subspace in the rotational spectrum and treat the molecular ensemble as a two-level system. Trapping molecules close to the transmission line surface allows a strong electric dipole coupling with the cavity photons with the effective coupling constant of $g_{\mathrm{eff}} = g\sqrt{N} \sim 10$ MHz [32].

Equation (19) yields that in the limit $a \gg c\nu$ the probability of atomic excitation with simultaneous emission of a photon into a single mode is given by $P \sim 4g_{\mathrm{eff}}^2/\omega^2$. For $g_{\mathrm{eff}} = 10$ MHz and $\omega = 1$ GHz we obtain $P \sim 10^{-4}$. If there are many (e.g., 100) cavity modes for which $a \gg c\nu$, then probabilities add up, and the "artificial" atom can get excited with the detectable probability $P \sim 10^{-2}$. Since the interference factor in Eq. (19) is governed by the atomic frequency $\omega$, the interference will not be washed out by summation over the field modes, and hence, spatial oscillations can be observed in this scheme.

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