

Let A be the area (the dependent variable) and r be the radius (the independent variable). As the radius changes from r to $r + dr$, the area changes from πr^2 to $\pi(r + dr)^2$. Therefore,

$$\begin{aligned}dA &= \pi(r + dr)^2 - \pi r^2 \\&= \pi[r^2 + 2r dr + (dr)^2] - \pi r^2 \\&= 2\pi r dr + \pi (dr)^2.\end{aligned}$$

The $\pi (dr)^2$ is called a “second order” term; we can neglect it because it’s very small (smaller than dr itself). Thus

$$dA = 2\pi r dr,$$

or

$$\frac{dA}{dr} = 2\pi r.$$