

Consider $\lim_{x \rightarrow 0^+} (x \ln x)$. This is an indeterminate form of the type $0 \cdot \infty$. To apply l'Hôpital's rule we must rewrite it as a quotient.

First try: $\lim_{x \rightarrow 0^+} \frac{x}{(\ln x)^{-1}}$ is an indeterminate form of type $\frac{0}{0}$. The rule gives

$$\lim_{x \rightarrow 0^+} \frac{1}{-\frac{1/x}{(\ln x)^2}} = \lim_{x \rightarrow 0^+} [-x(\ln x)^2],$$

which is more complicated than the original problem.

Second try: $\lim_{x \rightarrow 0^+} \frac{\ln x}{1/x}$ is an indeterminate form of type $\frac{\infty}{\infty}$. The rule gives

$$\lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2} = \lim_{x \rightarrow 0^+} (-x) = 0.$$

The second method works; the first one doesn't.