1. Center of mass

Suppose that, for $i = 1, 2, \ldots, N$, there is a particle of mass m_i located at position $\vec{\mathbf{r}}_i$. Then

$$M = \sum_{i=1}^{N} m_i$$

is the total mass. The **center of mass** is a vector (position) defined by

$$\overline{\vec{\mathbf{r}}} = \vec{\mathbf{r}}_{\rm cm} = \frac{\sum_{i=1}^{N} m_i \vec{\mathbf{r}}_i}{M} \,.$$

[Insert graphic of points labeled by masses and radius vectors. (For the experts, that should be "radii vectores".)]

This is a weighted average of the positions of the N particles. (If all the masses are equal, it is just the ordinary average of the positions —

$$\frac{\sum_{i=1}^{N} \vec{\mathbf{r}}_i}{N}$$

— the center of the particle cloud.)

In components, we have

$$\overline{x} = x_{\rm cm} = \frac{\sum_{i=1}^{N} m_i x_i}{M}$$

[Insert graphic of y–z plane seen on edge, accompanied by two particles, one on each side, with perpendicular distances labeled x_1 and $-x_2$.]

and similarly

$$\overline{y} = \frac{\sum_{i=1}^{N} m_i y_i}{M} \,, \qquad \overline{z} = \frac{\sum_{i=1}^{N} m_i z_i}{M} \,.$$

2. Moment of inertia

Now for something different, but similar ...

The **moment of inertia** of the particles **about** (or **around**) **the** z **axis**) is

$$I_z = \sum_{i=1}^{N} m_i (x_i^2 + y_i^2) = \sum_{i=1}^{N} m_i r_i^2.$$

[Insert graphic of z axis pointing out of the plane ($\odot$), with particles located at distances r_1 and r_2.]

$\frac{I_z}{M}$ is the weighted average of the squares of the distances r_i of the particles from the z axis. Similarly,

$$I_y = \sum_{i=1}^{N} m_i (z_i^2 + x_i^2), \qquad I_x = \sum_{i=1}^{N} m_i (y_i^2 + z_i^2).$$

It is up to you to read the physics textbook to learn what centers of mass and moments of inertia have to do with **total momentum**, **angular momentum**, and **kinetic energy**.