TEX document #2

The mass of the matter in a cube centered at

$$\vec{\mathbf{r}} = \langle x, y, z \rangle = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

is the sum of the masses of the particles in the cube. The average density in the cube is its mass over its volume:

$$\frac{M_{\text{cube}}}{\Delta x \Delta y \Delta z} = \frac{M_{\text{cube}}}{(\Delta x)^3}\,.$$

Take the limit as the cube shrinks to the point $\vec{\mathbf{r}}$:

$$\rho(\vec{\mathbf{r}}) = \lim_{\Delta x \to 0} \frac{M_{\text{cube at } \vec{\mathbf{r}}}}{(\Delta x)^3}\,.$$

This is **the density at $\vec{\mathbf{r}}$**.

Now we think of the total mass as the sum of the masses in all the cubes. In the limit of infinitesimal cubes it's

$$M = \iiint_E \rho(\vec{\mathbf{r}})\, dV = \iiint_E \rho(\vec{\mathbf{r}})\, dx\, dy\, dz,$$

where E is the region in 3-space occupied by the body.

[Ultimately should insert a link, "For more details click here", leading to a Riemann-sum discussion of this integral.]