Maclaurin Series of Familiar Functions

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots = \sum_j \frac{x^j}{j!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\frac{1}{1-x} = (1-x)^{-1} = 1 + x + x^2 + \cdots = \sum_j x^j \quad \text{(``geometric series'')}$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots = -\sum_j \frac{x^j}{j}$$

(corresponding to the Taylor series $\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \cdots$)

$$\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \cdots$$

(corresponding to the Taylor series $\sqrt{x} = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \cdots$)

$$(1+x)^{-\frac{1}{2}} = 1 - \frac{x}{2} + \frac{3}{8}x^2 - \cdots$$

And in general,

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \cdots \quad \text{(``binomial series'')}$$

which becomes a familiar finite polynomial when p is a positive integer: for example,

$$(1+x)^3 = 1 + 3x + 3x^2 + x^3 \quad \text{(exactly)}$$