

The differential equation satisfied by the angular position of an ideal, frictionless pendulum is

$$\frac{d^2\theta}{dt^2} + \sin\theta = 0. \quad (1)$$

If the pendulum is undergoing small oscillations around its equilibrium position (straight down,  $\theta = 0$ ), then  $\theta$  is always small, and we can replace  $\sin\theta$  by the first term in its Taylor expansion:

$$\frac{d^2\theta}{dt^2} + \theta = 0. \quad (2)$$

The approximate equation (2) is linear, and we know how to write down the solution!

For a better approximation, we could keep the second nonvanishing term in the Taylor expansion:

$$\frac{d^2\theta}{dt^2} + \theta - \frac{1}{6}\theta^3 = 0. \quad (3)$$

Although (3) is hard to solve exactly, it is slightly easier to deal with than (1) (for example, in constructing an approximate solution by numerical methods or by perturbation theory).