Problem: Find the first few terms of the Maclaurin series of

$$g(x) = \frac{1}{5-3x}\,.$$

Method 1: Apply Taylor's formula directly: Calculate $g(0)$, $g'(0)$, $g''(0)$, ... and construct

$$g(x) \approx g(0) + g'(0)x + \tfrac{1}{2}g''(0)x^2 + \cdots .$$

(This is probably the **worst** way to do the problem — certainly it's the most boring.)

Method 2: Treat this as a **long division** problem (see Stewart p. 662):

$$\begin{array}{r l}
 & \tfrac{1}{5} + \tfrac{3}{25}x + \cdots \\ \cline{2-2}
5-3x) & 1 + 0x + 0x^2 + \cdots \\
 & 1 - \tfrac{3}{5}x \\ \cline{2-2}
 & \quad\;\; \tfrac{3}{5}x \\
 & \quad\;\; \tfrac{3}{5}x - \tfrac{9}{25}x^2 \\ \cline{2-2}
 & \qquad\qquad \tfrac{9}{25}x^2 \cdots
\end{array}$$

Method 3: Write $\frac{1}{5-3x}$ as $\frac{1}{5}\,\frac{1}{1-\frac{3}{5}x}$. Now apply the **geometric series**

$$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \cdots$$

with $z = \frac{3}{5}x$.

Method 4 (the method marketed by our sponsor)**:** We are seeking a solution for the equation

$$(5-3x)g(x) = 1.$$

Assume that

$$g(x) = a_0 + a_1 x + a_2 x^2 + \cdots .$$

Then

$$\begin{aligned} 1 &= (5-3x)(a_0 + a_1 x + a_2 x^2 + \cdots) \\ &= 5a_0 + 5a_1 x - 3x_0 x + 5a_2 x^2 - 3a_1 x^2 + \cdots . \end{aligned}$$

After we combine terms, the total coefficient of each power x^j must match the corresponding coefficient on the other side of the equation. This principle gives a sequence of **recursion relations**,

$$\begin{aligned} 1 &= 5a_0 \, , && (x^0) \\ 0 &= 5a_1 - 3a_0 \, , && (x^1) \\ 0 &= 5a_2 - 3a_1 \, , && (x^2) \\ &= \ldots . \end{aligned}$$

These can be solved in succession for the coefficients:

$$\begin{aligned} a_0 &= \tfrac{1}{5}, \\ a_1 &= \tfrac{3}{5}a_0 = \tfrac{3}{25}, \\ a_2 &= \tfrac{3}{5}a_1 = \tfrac{9}{125}, \\ &= \ldots . \end{aligned}$$

It's easy to see that the general solution is

$$a_{j+1} = \frac{3^{j+1}}{5^{j+2}} \, .$$