Problem: Expand $\sin(2x+1)$ around $x=0$.

THE WRONG WAY: We know that

$$\sin z = z - \frac{z^3}{6} + \frac{z^5}{120} + \cdots,$$

so it seems natural to substitute $2x+1$ for z:

$$\sin(2x+1) = (2x+1) - \frac{(2x+1)^3}{6} + \frac{(2x+1)^5}{120} + \cdots.$$

Next, trying to combine terms, we multiply everything out and get

$$1 + 2x - \frac{1}{6}(1 + 6x + 12x^2 + 8x^3) + \frac{1}{120}(1 + 10x + \cdots) + \cdots$$

$$= (1 - \frac{1}{6} + \frac{1}{120} + \underbrace{\cdots}_{?}) + x(2 - 1 + \frac{1}{12} + \cdots) + \cdots.$$

This is a disaster! Even the very first term (the constant term) depends on the terms of **arbitrarily high order** in the Taylor series of the sine. The point is that this method of finding the Taylor series of the composite function won't work **unless the expression substituted for** z **approaches** 0 **as** $x \to 0$.

A RIGHT WAY: Note that $2x+1$ approaches 1, not 0, as x approaches 0. Therefore, we should be using the Taylor expansion of $\sin z$ around $z=1$:

$$\sin z = \sin(1) + \cos(1)\,(z-1) - \frac{1}{2}\sin(1)\,(z-1)^2 + \cdots.$$

Then $\sin(2x+1)$ can be successfully approximated for x near 0 by substituting $2x+1$ for z — that is, $2x$ for $z-1$.

$$\sin(2x+1) = \sin(1) + 2\cos(1)\,x - 2\sin(1)\,x^2 + \cdots.$$

(Of course, we can't find exact numerical values of $\sin(1)$ and $\cos(1)$, but that's life; they are the correct coefficients in this series.)