Taylor's Theorem, Version 2

If all the derivatives of the function f up through $f^{(N+1)}$ exist in an interval I containing the number a, then for all x in I,

$$f(x) = T_N(x) + R_N(x),$$

where T_N is the Nth-degree Taylor polynomial,

$$T_N(x) = \sum_{j=0}^{N} \frac{f^{(j)}(a)}{j!}(x-a)^j \,,$$

and there is some number z strictly between* a and x such that

$$R_N(x) = \frac{f^{(N+1)}(z)}{(N+1)!}(x-a)^{N+1} \,.$$

---

* "Strictly between" means that either $a < z < x$ or $x < z < a$, whichever is appropriate. Here is a fine point: If $x = a$, then there is no number strictly between them, so the theorem as stated is false. In that case, however, $f(x)$ is **exactly** equal to $T_N(x)$ (all of whose terms are zero except (possibly) the first ($j = 0$)).