Taylor's Theorem, Version 1

If all the derivatives of the function f up through $f^{(N+1)}$ exist in an interval I containing the number a, then for all x in I, $f(x)$ is well approximated by its Nth-degree Taylor polynomial,

$$T_N(x) = \sum_{j=0}^{N} \frac{f^{(j)}(a)}{j!}(x-a)^j \ ,$$

in the following sense:

$$f(x) = T_N(x) + R_N(x),$$

where

$$|R_N(x)| \leq \frac{M|x-a|^{N+1}}{(N+1)!}$$

with

$$M = \max_{z \in I} |f^{(N+1)}(z)|.$$