Welcome to Section 601 of Math 689, Polynomials and Polynomial Inequalities
Fall, 2013, MWF 01:50 – 02:40 pm, Room - TBA

About your instructor:

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Office Hours (in Milner 308):

• Tuesday 10:30 – 12:00 am, Thursday 10:30 – 12:00 am, and by appointment.

Textbook (Required Purchase):

• *Polynomials and Polynomial Inequalities*, by P. Borwein and T. Erdélyi; published by Springer, New York.

Web Page for Math 689:

• http://www.math.tamu.edu/∼terdelyi/689page

Course Description:

• Credit 3. Polynomials pervade mathematics, and much that is beautiful in mathematics is related to polynomials. Virtually every branch of mathematics, from algebraic number theory and algebraic geometry to applied analysis, Fourier analysis, and computer science has its corpus of theory arising from the study of polynomials. Historically, questions relating to polynomials, for example, the solution to polynomial equations, gave rise some of the most important problems of the day. The subject is now much too large to attempt an encyclopedic coverage even in a period of several years.

• The course intends to cover a large part of my Springer Graduate Text entitled "Polynomials and Polynomial Inequalities" (1995) written jointly with Peter Borwein as well as some more recent related research papers. This book is about polynomials and rational functions. The primary focus is on analytic properties of algebraic and trigonometric polynomials of a single variable, though many other topics are touched on. Topics include: the geometry of polynomials, orthogonal polynomials, Chebyshev and Markov systems, Müntz systems and Müntz-type density theorems, exponential sums; inequalities for polynomials and rational functions; extremal problems for various classes of constrained polynomials. The course assumes only an undergraduate familiarity with real and complex analysis (by passing at least Math 407 and 409 with an "A" at Texas A&M).

• Exercises constitute more than half of the book. They are provided with copious hints and sometimes complete outlines, much in the style of the classic treatise "Problems and Theorems in Analysis" by Pólya and Szegő. Students are encouraged to work on some of these problems they find the most interesting, and may ask for additional hints if necessary.

Course Objective:

• Students will be introduced to a beautiful part of mathematics and have a taste of classical real and complex analysis focusing mainly on polynomials of one complex variable. By the end of the course students are expected to have a better understanding of what research is about on a field as simple as "polynomials and polynomial inequalities".

Course Grade:

• The course grade will be given based on the quality of the student's presentation of certain research topics. These topics will be assigned in the beginning of the semester, and in the second half of the semester students will be expected to play the role of the lecturer and present the assigned material (following the weekly schedule) in a convincing fashion to the rest of the class. Students will also be requested to work on certain problems related to the course material and submit their work at the end of the semester. The weight of this in the course grade will be specified in the beginning of the semester.

Course Schedule:

• The schedule below is quite ambitious. It depends heavily on the common background and the number of the students how much of it, at what pace, and in what depth can be covered.

• Week 1: Chebyshev polynomials and some of their applications.

• Week 2: Basic polynomial inequalities: Bernstein-Szegő, Markov, Schur, Remez inequalities, Bernstein-type inequalities for rational functions.

• Week 3: Polarization inequalities on the unit circle, Basic properties of Chebyshev systems and Chebyshev spaces, Chebyshev polynomials in a Chebyshev Chebyshev spaces, Markov systems.

• Week 4: Descartes systems, Pinkus-Smith improvement theorem for Descartes systems, Müntz spaces.

• Week 5: Müntz's theorem, variations on Müntz's theorem, Müntz-type density problems, "full Müntz theorem".

• Week 6: Bounded Remez-type inequality for non-dense Müntz spaces, Müntz's theorem on compact subsets of $[0,\infty)$, a solution to Newman's product problem.

• Week 7: Inequalities for Müntz polynomials and exponential sums. Newman's Markov-type inequality for Müntz polynomials, Bernstein-type inequalities for exponential sums.

• Week 8: Markov and Bernstein type inequalities for polynomials with restricted zeros.

• Week 9: Inequalities for polynomials with constrained coefficients, Konyagin's proof of Littlewood's conjecture on the L_1 norm of Littlewood polynomials.

• Week 10: Location and distribution of the zeros of polynomials with constrained coefficients, the Erdős-Turán theorem, and some more recent results. Lorentz representation and Lorentz degree of polynomials positive on $[-1,1]$.

• Week 11: Flat and ultraflat sequences of unimodular polynomials. A solution to Saffari's phase problem.

• Week 12: Rudin-Shapiro polynomials. The size of Fekete polynomials (polynomials formed with Legendre symbol coefficients) on the unit circle.

• Week 13: Mahler measure of polynomials. The size of cyclotomic Littlewood polynomials on the unit circle.

• Week 14: A solution to Littlewood's Problem 22. Alternatively, pseudo-Boolean functions, the Coppersmith-Rivlin inequality and some of its applications.

Scholastic dishonesty will not be tolerated:

• Any instance of scholastic dishonesty will be handled as consistent with University Regulations.

Copyright Statement:

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Americans with Disabilities Act (ADA) Policy Statement:

• The Americans with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring an accommodation, please contact the Department of Student Life, Services for Students with Disabilities, in Room 126 of the Koldus Building or call 845-1637.

Academic Integrity Statement:

• http://www.tamu.edu/aggiehonor