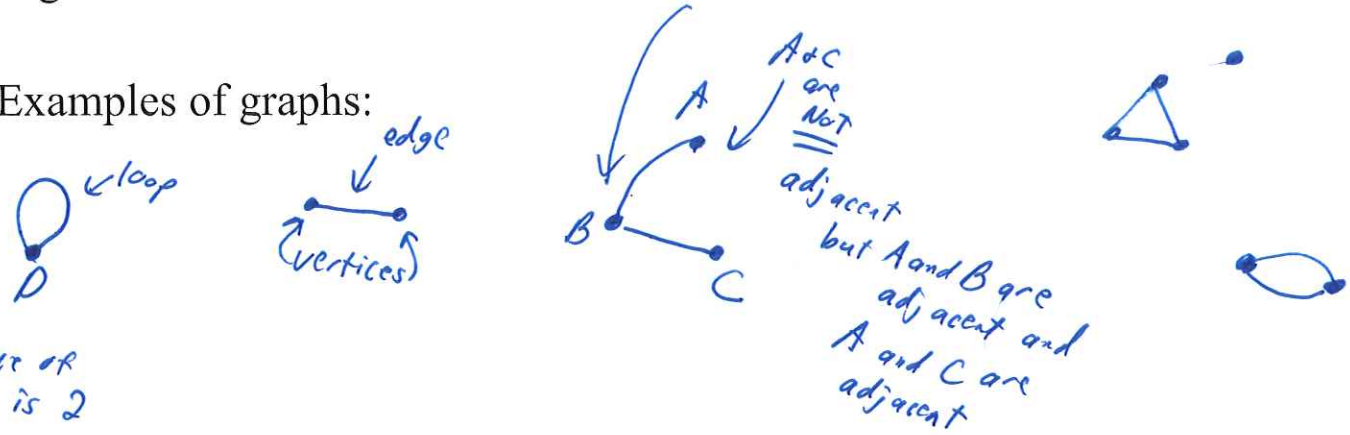
9/8/15 week in Review Math 167

CHAPTER 1 – URBAN SERVICES

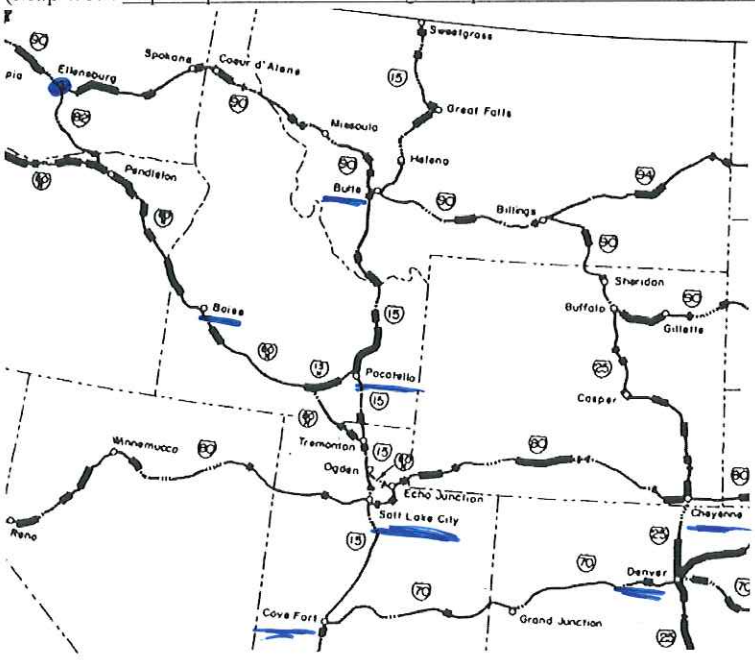
A ***graph*** is a collection of one or more points (***vertices***). ***Edges*** connect vertices. A ***loop*** is an edge that connects a vertex to itself. A ***simple graph*** contains no loops. Two different vertices are ***adjacent*** if they are connected by an edge. The ***valence*** or ***degree*** of a vertex is the number of edges at the vertex.

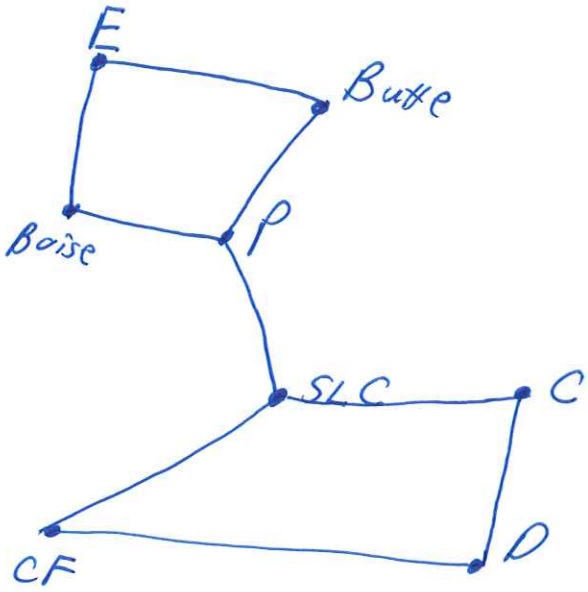
Examples of graphs:



Use the map below to represent Ellenburg, Butte, Boise, Pocatello, Salt Lake City, Cheyenne, Denver, and Cove Fort and the main roads that connect them as a simple graph. What cities are adjacent to Salt Lake City?

(Map from https://upload.wikimedia.org/wikipedia/commons/7/70/Interstate_Highway_status_unknown_date.jpg)

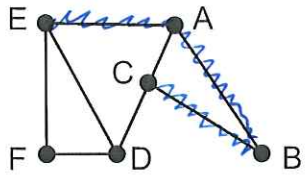




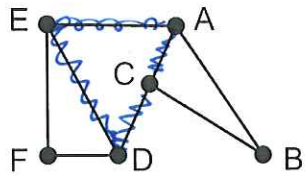
Adj to SLC are P, C, CF

A ***path*** is a connected sequence of edges showing a route on a graph. It is named using the list of adjacent vertices that create the edges.
A path that uses every edge exactly once is an ***Euler path***.
If the path ends at the same vertex it started at, it is a ***circuit***.
A circuit that uses every edge exactly once is an ***Euler circuit***.

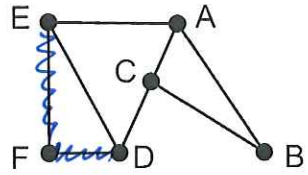
Classify the list of vertices for the graphs below with the most precise term that is appropriate.



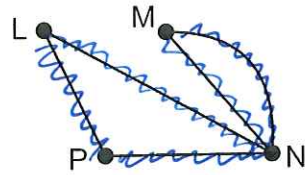
EABC path (but not an Euler path)



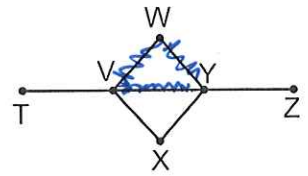
AEDCA Circuit (but not an Euler circuit)



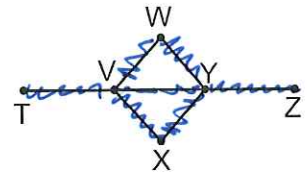
EFDB Not a path



NMNLPN Euler circuit



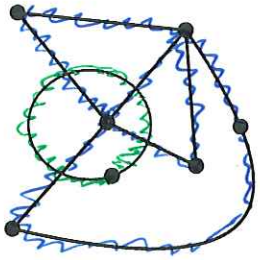
VWYV Circuit (but not an Euler circuit)



TVWYVXYZ Euler path

A graph is ***connected*** if for every pair of vertices there is a path that connects them.

Is the graph below connected? If not, how many components (sub-graphs) are there? Note that a component could consist of a vertex or vertices connected by edges

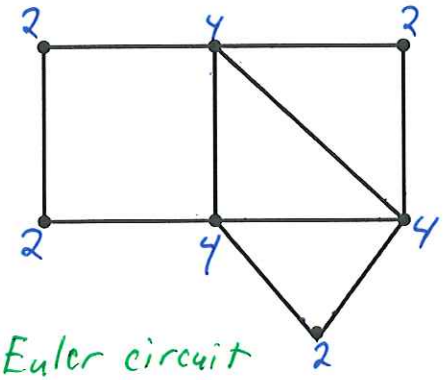


NOT connected
2 components

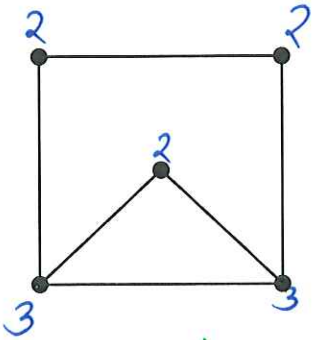
Euler's Theorem for a connected graph

1. If the graph has no vertices of odd degree, then it has at least one Euler circuit and if a graph has an Euler circuit, then it has no vertices of odd degree.
2. If a graph has 2 vertices of odd degree, then there is at least one Euler path, but no Euler circuit. Any Euler path must start at a vertex with an odd degree and end at the other vertex of odd degree.
3. If the graph has more than two vertices of odd degree, then it does not have an Euler path.

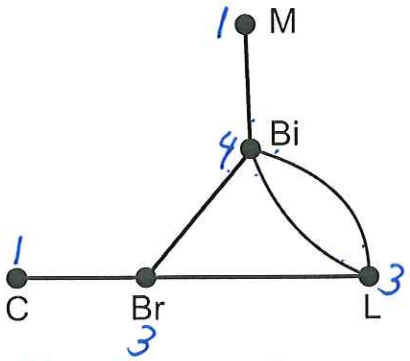
Determine whether the following graphs contain an Euler path, or Euler circuit, or neither. Count valence



Euler circuit
b/c all valences are even



Euler path
b/c 2 valences are odd

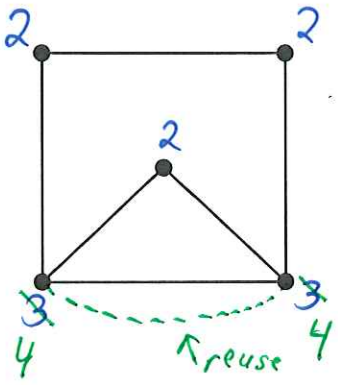


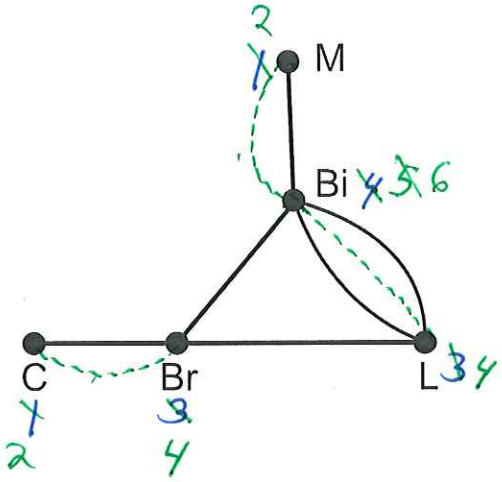
No Euler path or Euler circuit
b/c more than 2 vertices have odd valence

Chinese postman problem: Cover all the edges at least once with the minimum cost.

When you ***Eulerize*** a graph, you reuse edges as necessary to form an Euler circuit.

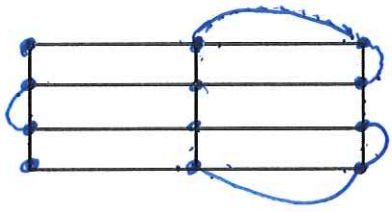
Count valences



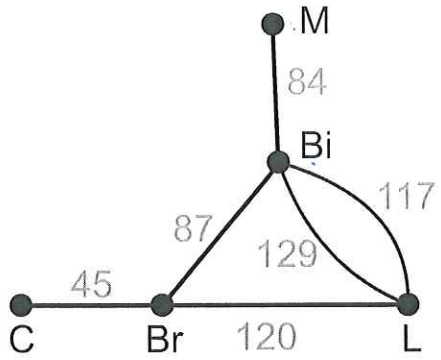


If a graph is ***rectangular***, it is a group of rectangular blocks that form a larger rectangle.

One way to Eulerize a rectangular graph is to use an ***edge-walker algorithm***. Starting at one vertex of the outer rectangle, reuse edges to each odd vertex that connects to the next vertex.



Edges of a graph may have an associated cost for traversing the edge. The graph has the distance between the cities in miles. What is the cost to go from Bristol to Manchester?



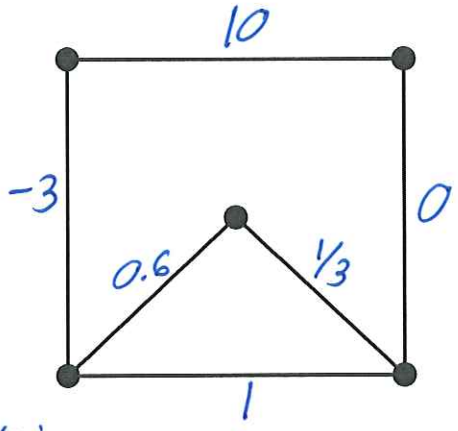
cheaper

Br —87— Bi —84— M = 171 miles

Br —120— L —117— Bi —84— M = 321 miles

chose cheaper one

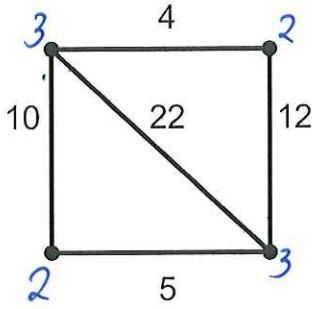
The cost of an edge may have any value – just watch the units given.



Make into Euler circuit

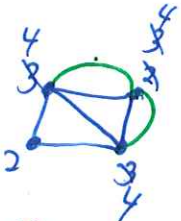
Eulerize the graph below at the lowest cost. The cost of an edge is the time to travel between the vertices in minutes.

count valences



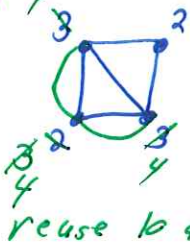
reuse 22 min

Total cost
4+12+5+10+22
+22
= 75 min



reuse 4 and 12

Total cost
4+12+5+10+22
+4+12
= 69 min

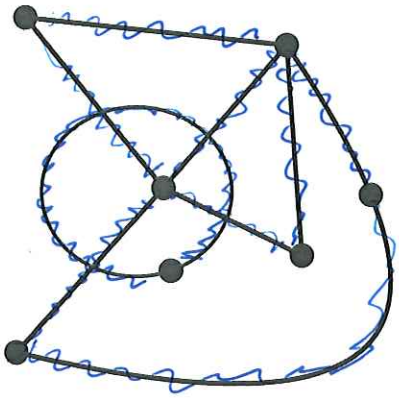


reuse 10 and 5

Total cost of
4+12+5+10+22
+10+5
= 68 min

SAMPLE EXAM QUESTIONS FROM CHAPTER 1

1. Mark all true statements about the graph on the right:

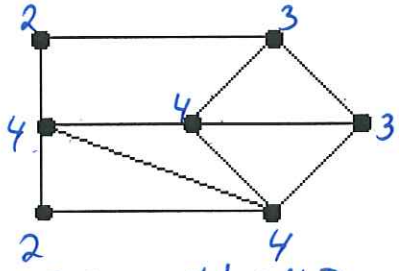


7 vertices
9 edges

(A) The graph is connected. F
(B) The graph has 7 vertices. T
(C) The graph has 8 edges. F
(D) The graph has 9 vertices F
(E) None of these statements are true.

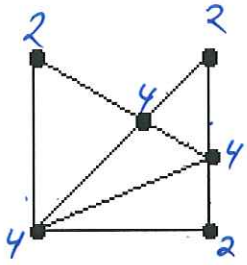
2. Which of the graphs below have Euler circuits? Count valences

I



Euler path but NOT and Euler circuit

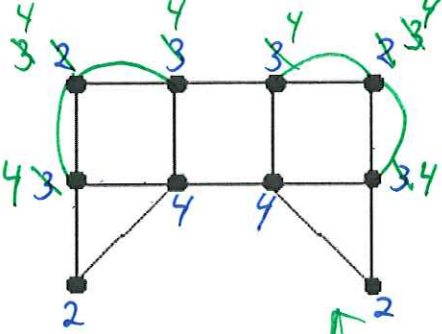
II

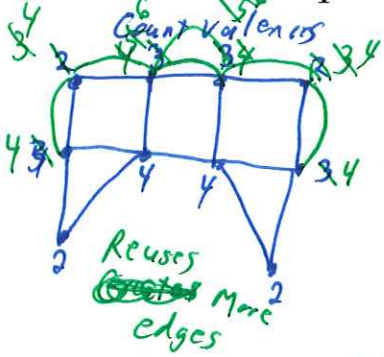


Euler circuit

(A) Only graph I
(B) Only graph II
(C) Both graph I and graph II
(D) Neither graph have an Euler circuit
(E) Need more information to determine the answer

3. In order to Eulerize the graph below, give the fewest number of edges that need to be duplicated.

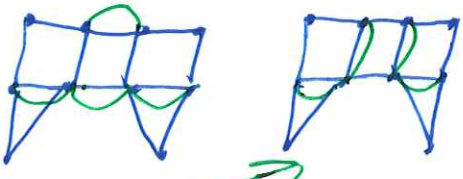




Count valences

Reuses ~~Excess~~ More edges

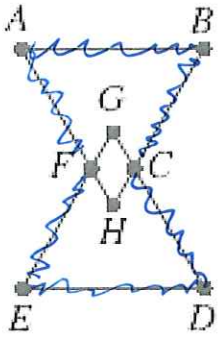
Each reused 4 roads



(A) More than 4
(B) 4
(C) 3
(D) 2
(E) 1

4. Classify the following: ABCDEFA.

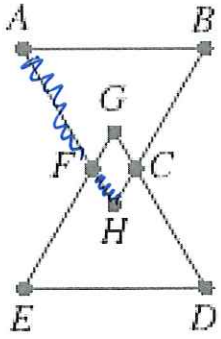
(A) Not a path
(B) A path
(C) A circuit
(D) An Euler path
(E) An Euler circuit



5. Classify the following: AFHGCB.

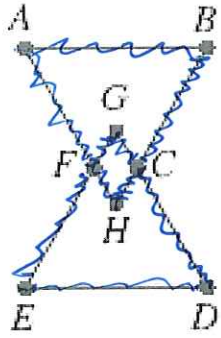
(A) Not a path
(B) A path
(C) A circuit
(D) An Euler path
(E) An Euler circuit

not adjacent

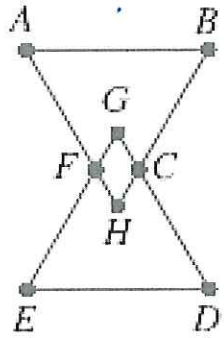


6. Classify the following: ABCGFHCDEFA.

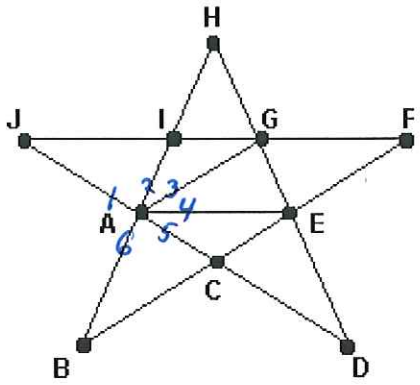
(A) Not a path
(B) A path
(C) A circuit
(D) An Euler path
(E) An Euler circuit



7. How many edges does the graph have? 10



8. What is the valence of vertex A in the graph below? 6



9. After a major natural disaster, such as a flood, hurricane, or tornado, many tasks need to be completed as efficiently as possible. For which situation below would finding an Euler circuit or an efficient Eulerization of a graph be the appropriate mathematical technique to apply?

(A) The electric company must check several substations for malfunctions. ↑ vertices

(B) The gas company must check along all gas lines for possible leaks.

(C) The phone company must respond to customers' needs in several parts of town.

(D) The water company must spot-check the integrity of eight water towers located throughout the city.