

CHAPTER 9 – VOTING

Majority Rule: Each voter votes for one candidate. The candidate with the majority of the votes wins. Majority means MORE than half.

Plurality Method: Each voter votes for one candidate. The candidate with the most votes is the winner.

Condorcet's Method: Each voter ranks the candidates (preference list voting). Each candidate is compared to each of the other candidates. If a candidate wins all of his/her one-on-one contests (is undefeated), he/she is declared the Condorcet winner.

Borda Count: Each voter ranks the n candidates with $n - 1$ points assigned to the first choice, $n - 2$ to the second choice and so on. The candidate with the most points wins. Other rank methods use different point values.

Hare System: If there is no majority winner, then the candidate(s) with the fewest number of first place votes is(are) eliminated and the results are calculated again. If there is still no majority winner, the process continues until a majority winner is found or the remaining candidates are tied.

Sequential Pairwise Voting: Candidates are compared two at a time in a predetermined order known as an agenda. The winner of the pairing is compared to the next candidate on the pre-determined list. This process continues until a winner is determined.

Approval Method: Each voter votes for all the candidates they approve of. The candidate with the most votes wins.

For all methods, a tie-breaking mechanism should be in place prior to the election. Methods could include flipping a coin, drawing straws, number of first place votes, introducing an additional voter, and other methods.

Example

Assume that the following list reflects the voting preferences of all voters.

Pref. List	ADCB	ABCD	BCDA	BCAD	CBDA	CDBA	DCBA
# Voters	2	3	4	5	4	3	1

(a) Who is the majority winner?

Candidate	1 st place votes
A	2 + 3 = 5
B	4 + 5 = 9
C	4 + 3 = 7
D	1 = 1

$2+3+4+5+4+3+1 = 22$ voters

so need more than $\frac{22}{2} = 11$ votes to win

No one has more than 11 votes, so no majority winner

(b) Who is the plurality winner?

B had the most votes, so B is plurality winner

(c) Find the winner using the Hare system.

Pref. List	ADCB	ABCD	BCDA	BCAD	CBDA	CDBA	DCBA
# Voters	2	3	4	5	4	3	1

Candidate	Votes in 1 st round	Votes in 2 nd round	Votes in 3 rd round
A	2 + 3 = 5	= 5	E eliminate
B	4 + 5 = 9	= 9	+ 3 = 12
C	4 + 3 = 7	+ 1 = 8	+ 2 = 10
D	1 = 1	E eliminate	

still no majority winner

B wins using Hare System b/c now has more than half the votes

Pref. List	ADCB	ABCD	BCDA	BCAD	CBDA	CDBA	DCBA
# Voters	(2)	(3)	(4)	(5)	(4)	(3)	(1)

(d) Who is the Condorcet winner? **B** b/c B is undefeated

Choices	Votes	Choices	Votes	Winner
A over B	2 + 3 = 5	B over A	4 + 5 + 4 + 3 + 1 = 17	B
A over C	2 + 3 = 5	C over A	4 + 5 + 4 + 3 + 1 = 17	C
A over D	2 + 3 + 5 = 10	D over A	4 + 4 + 3 + 1 = 12	D
B over C	3 + 4 + 5 = 12	C over B	2 + 4 + 3 + 1 = 10	B
B over D	3 + 4 + 5 + 4 = 16	D over B	2 + 3 + 1 = 6	B
C over D	3 + 4 + 5 + 4 + 3 = 19	D over C	2 + 1 = 3	C

(e) Who is the sequential pairwise winner with the agenda ABCD? **B**
 Using work from Condorcet ...



Pref. List	ADCB	ABCD	BCDA	BCAD	CBDA	CDBA	DCBA
# Voters	(2)	(3)	(4)	(5)	(4)	(3)	(1)

(f) Who is the Borda count winner? **C**
 # candidates - 1

	1 st place * 3 pts	2 nd place * 2 pt	3 rd place * 1 pts	Total
A	2 + 3 = 5		5 = 5	5(3) + 0(2) + 5(1) = 20
B	4 + 5 = 9	3 + 4 = 7	3 + 1 = 4	9(3) + 7(2) + 4(1) = 45
C	4 + 3 = 7	4 + 5 + 1 = 10	2 + 3 = 5	7(3) + 10(2) + 5(1) = 46
D	1 = 1	2 + 3 = 5	4 + 4 = 8	1(3) + 5(2) + 8(1) = 21

Example

Use the chart below to determine what kind of game will be played if each player marks all the games he approves of and the approval method is used to determine the winner.

Game played is Trivial Pursuit

	A	B	C	D	E	F	G	H	# Approvals
Pictionary		x			x			x	3
Scrabble	x		x		x		x		4
Dominos		x	x		x			x	4
Trivial Pursuit	x		x	x	x			x	5 ←
Twister			x		x	x			3

Example

A class of 31 students wanted to elect two people to represent them at a meeting. They decided to use the approval method. Use the chart below to determine which two people will be elected. The top row lists the number of voters who approved of the candidate combination in that column

	4	8	5	6	4	1	3	
Jeanetta	x			x			x	$4+6+3=13$
Mittie		x	x	x			x	$8+5+6+3=22$ 1 st place
Wilton	x		x		x		x	$4+5+4+3=16$ 2 nd place
Jamaal	x	x						$4+8=12$
Yong		x	x			x		$8+5+1=14$

People chosen are Mittie and Wilton

Example

Twenty-seven board members vote on four candidates, A, B, C, or D, for a new position on their board. Their preference schedules are shown below.

Pref. List	ABCD	DABC	BCDA	CBDA
# Voters	6	8	7	4

(a) Who is the majority winner? *More than half* **No one**

Candidate	1 st place votes
A	6
B	7
C	4
D	8

Need MORE than half the votes, so need MORE than $\frac{6+8+7+4}{2} = \frac{25}{2} = 12.5$ votes to win majority

(b) Who is the plurality winner? *most* **D** has most votes

(c) Find the winner using the Hare system. **B**

Pref. List	ABCD	DABC	BCDA	CBDA
# Voters	6	8	7	4

Candidate	Votes in 1 st round	Votes in 2 nd round	Votes in 3 rd round
A	6	= 6	Eliminate
B	7	+4 = 11	+6 = 17
C	4	Eliminate	—
D	8	= 8	= 8

Still no majority

B has majority after C & A are eliminated

Math 167 Ch 9 Week in Review

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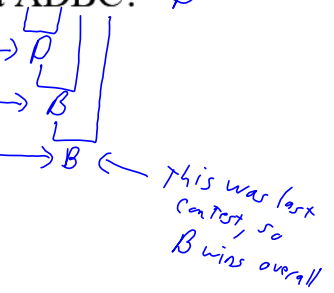
Pref. List	ABCD	DABC	BCDA	CBDA
# Voters	6	8	7	4

(d) Who is the Condorcet winner? *No one was undefeated*

Choices	Votes	Choices	Votes	Winner
A over B	6 + 8 = 14	B over A	7 + 4 = 11	A
A over C	6 + 8 = 14	C over A	7 + 4 = 11	A
A over D	6 = 6	D over A	8 + 7 + 4 = 19	D
B over C	6 + 8 + 7 = 21	C over B	4 = 4	B
B over D	6 + 7 + 4 = 17	D over B	8 = 8	B
C over D	6 + 7 + 4 = 17	D over C	8 = 8	C

(e) Who is the sequential pairwise winner with the agenda ADBC? *B*

when A played D, D won, so D continues
 when D played B, B won, so B continues
 when B played C, B won, so B continues



Pref. List	ABCD	DABC	BCDA	CBDA
# Voters	6	8	7	4

(f) Who is the Borda count winner? *B had most points*

*4-1
4
4
4
4
candidates*

	1 st place * 3 pts	2 nd place * 2 pt	3 rd place * 1 pts	Total
A	6	8		6(3) + 8(2) + 0(1) = 34
B	7	6 + 4 = 10	8	7(3) + 10(2) + 8(1) = 49
C	4	7	6	4(3) + 7(2) + 6(1) = 32
D	8		7 + 4 = 11	8(3) + 0(2) + 11(1) = 35

Sample exam questions would resemble the problems we worked, but may not include the charts.