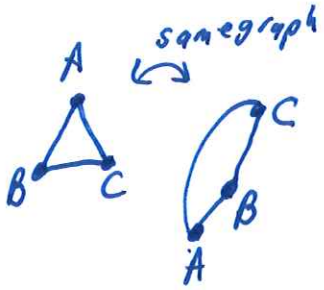
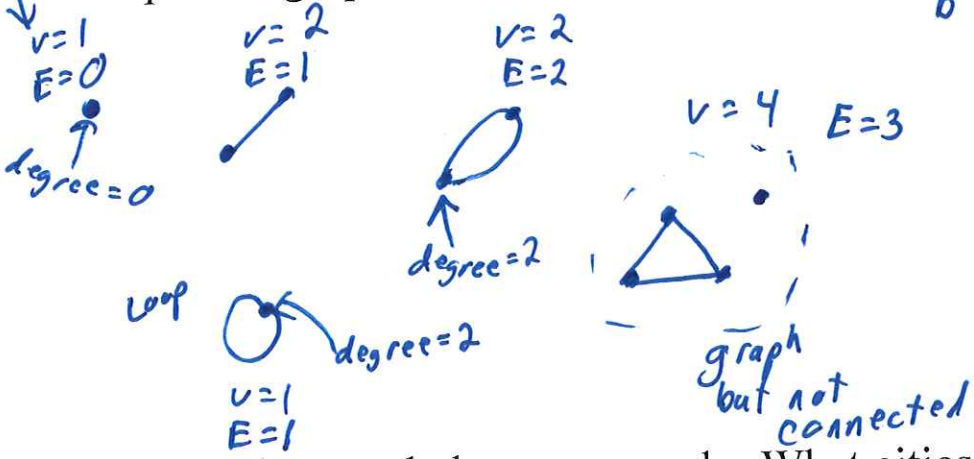
CHAPTER 1 – URBAN SERVICES

A ***graph*** is a collection of one or more points (***vertices***). The vertices may be connected by ***edges***. Two vertices are ***adjacent*** if they are connected by an edge. The ***valence*** or ***degree*** of a vertex is the number of edges at the vertex.



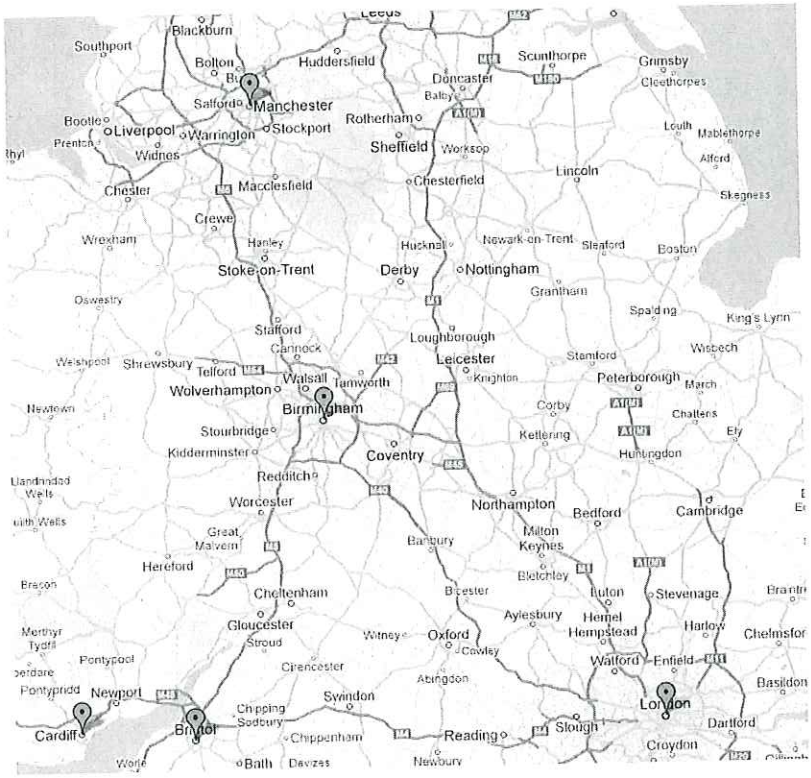
vertices

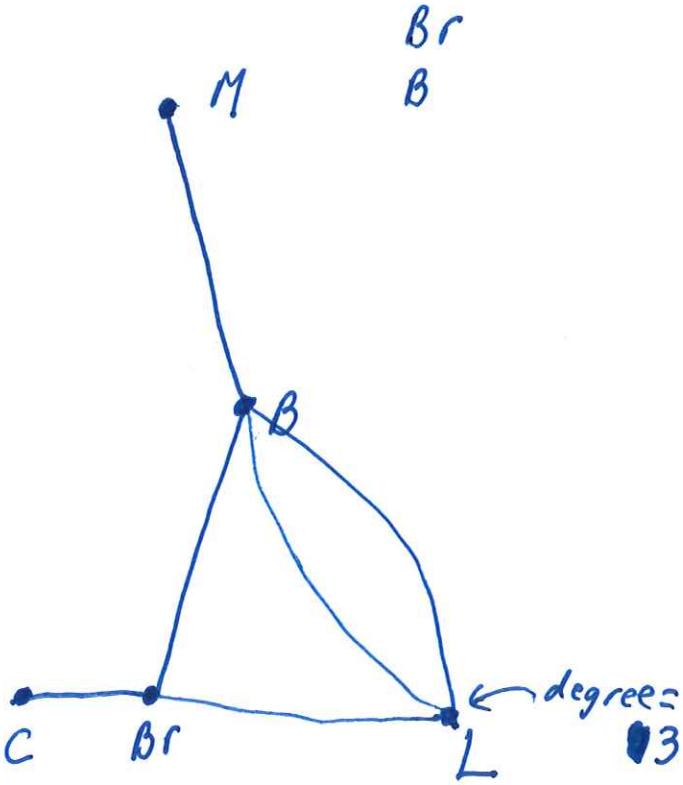
Examples of graphs:



Represent the map below as a graph. What cities are adjacent to London?

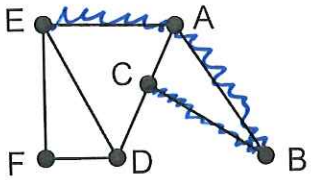
use main roads



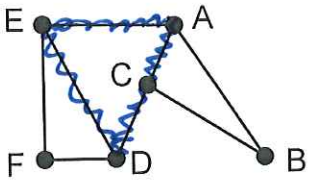


A ***path*** is a connected sequence of edges showing a route on a graph that starts and ends at a vertex. ← does not have to be the same vertex
A path that uses every edge exactly once is an ***Euler path***.
If the path ends at the same vertex it started at, it is a ***circuit***.
A circuit that uses every edge exactly once is an ***Euler circuit***.

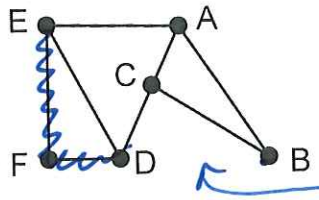
Classify the list of vertices for the graphs below



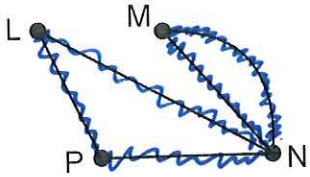
EABC path but not an Euler path



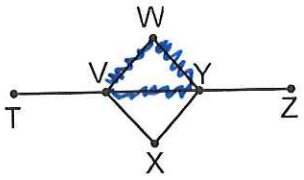
AEDCA Circuit but not an Euler circuit



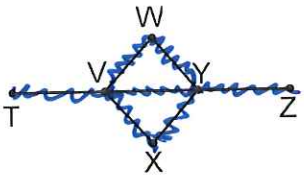
EFDB Not a path
not adjacent



NMNLPN Euler circuit



VWYV Circuit but not an Euler circuit

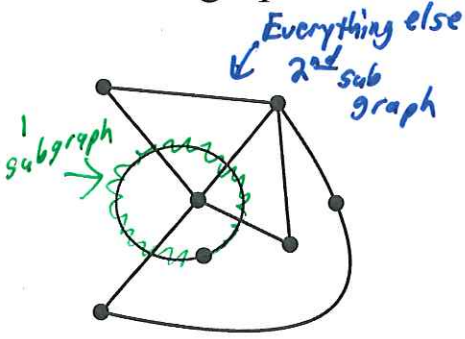


TVWYVXYZ Euler path

A graph is ***connected*** if for every pair of vertices there is a path that connects them.

components

Is the graph below connected? If not, how many sub-graphs are there?



Not connected

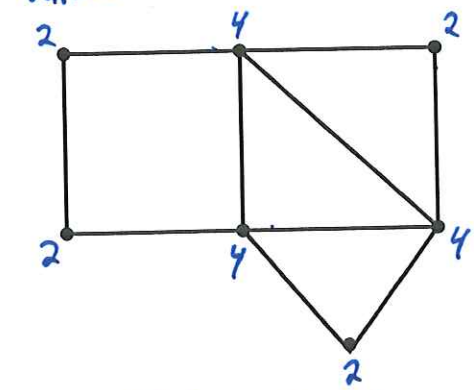
2 subgraphs

Euler's Theorem for a connected graph

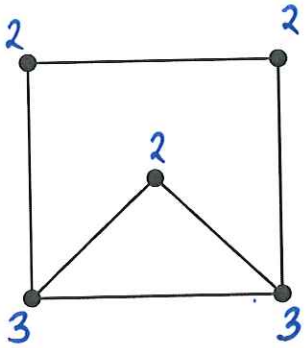
1. If the graph has no vertices of odd degree, then it has at least one Euler circuit and if a graph has an Euler circuit, then it has no vertices of odd degree.
2. If a graph has 2 vertices of odd degree, then there is at least one Euler path, but no Euler circuit. Any Euler path must start at a vertex with an odd degree and end at the other vertex of odd degree.
3. If the graph has more than two vertices of odd degree, then it does not have an Euler path.

All are connected

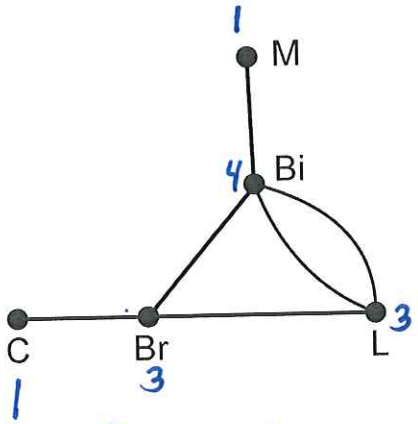
Count valence



Euler circuit b/c all valences are even



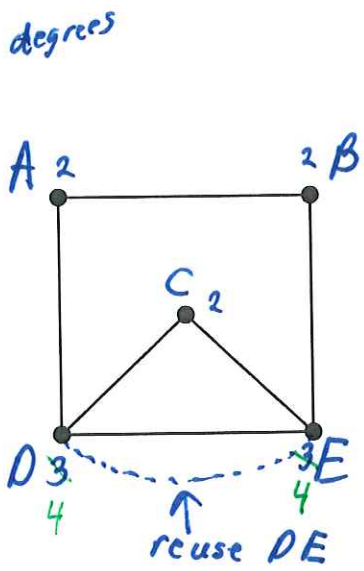
Euler path b/c 2 vertices with odd degree

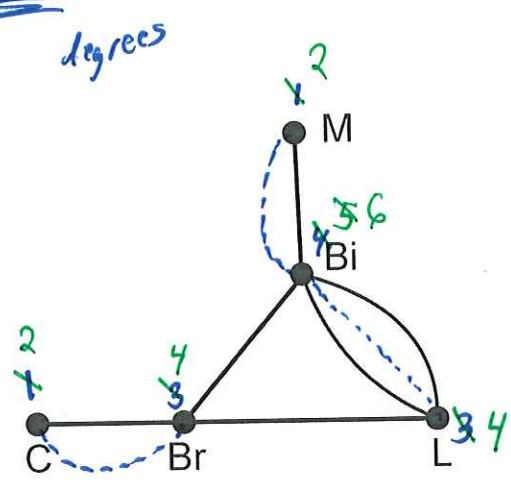


No Euler path or Euler circuit b/c more than 2 vertices with odd degree

Chinese postman problem: Cover all the edges at a minimum cost.

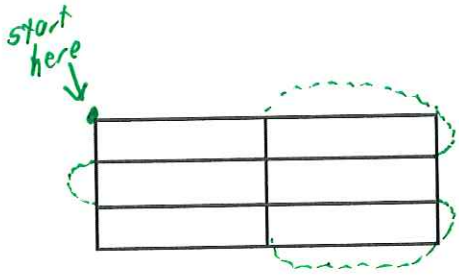
When you ***Eulerize*** a graph, you reuse edges to make an Euler circuit.



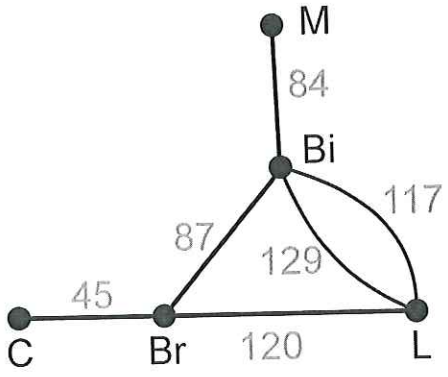


If a graph is ***rectangular***, it is a group of rectangular blocks that form a larger rectangle.

One way to Eulerize a rectangular graph is to use an ***edge-walker algorithm***. Starting at one vertex of the outer rectangle, ~~add~~ reuse edges to each odd vertex that connects to the next vertex.



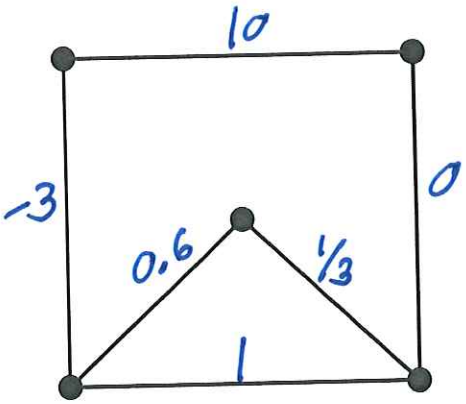
Edges of a graph may have an associated cost for traversing the edge. The graph has the distance between the cities in miles. What is the cost to go from Bristol to Manchester?



Br $\xrightarrow{87}$ Bi $\xrightarrow{84}$ M = 171 miles

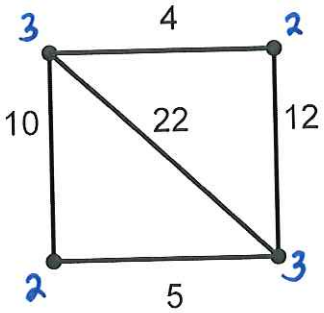
Br $\xrightarrow{120}$ L $\xrightarrow{117 \text{ or } 129}$ Bi $\xrightarrow{84}$ M = 321 or 333 mi

The cost of an edge may have any value – just watch the units given.



Eulerize the graph below at the lowest cost. The cost of an edge is the time to travel between the vertices in minutes.

degree



reuse 22 min

Total of 75 min

reuse 16 min

Total of 69 min

reuse 15 min

Total of 68 min

SAMPLE EXAM QUESTIONS FROM CHAPTER 1

Mark all true statements about the graph on the right:

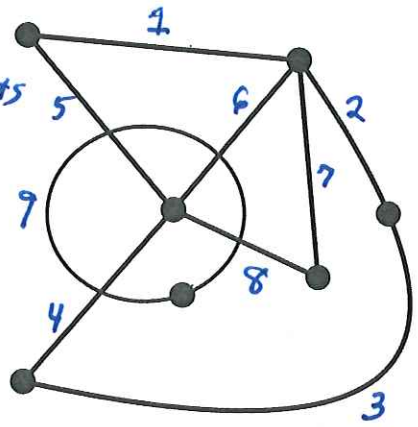
(A) The graph is connected. F 2 components

(B) The graph has 7 vertices.

(C) The graph has 8 edges. F 9 edges

(D) The graph has 9 vertices

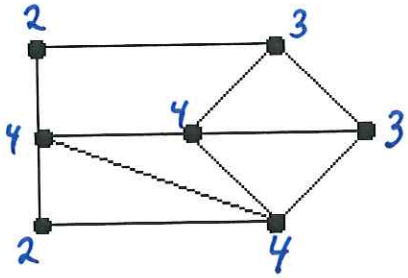
(E) None of these statements are true.



Which of the graphs below have Euler circuits?

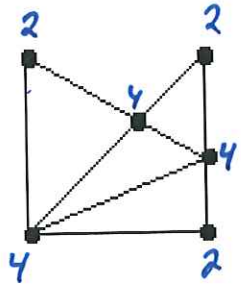
valence

I



Euler Path but not Euler Circuit

II



Euler circuit

(A) Only graph I

(B) Only graph II

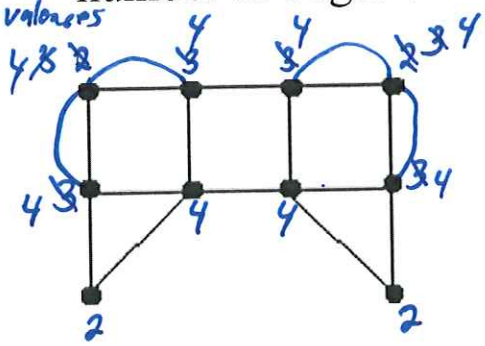
(C) Both graph I and graph II

(D) Neither graph have an Euler circuit

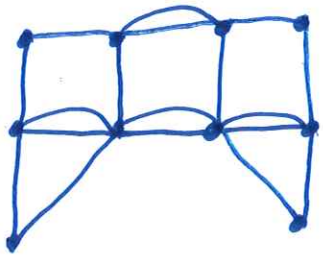
(E) Need more information to determine the answer

In order to eulerize the graph below, give the fewest number of edges that need to be added or duplicated.

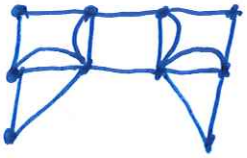
valences



or



or



(A) More than 4

(B) 4

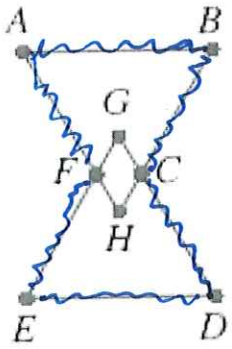
(C) 3

(D) 2

(E) 1

Classify the following: ABCDEFA.

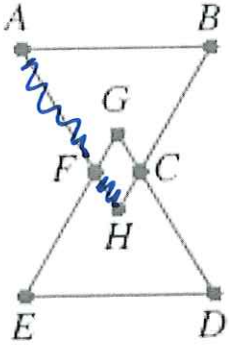
(A) Not a path
(B) A path
(C) A circuit
(D) An Euler path
(E) An Euler circuit



Classify the following: AFHGCB.

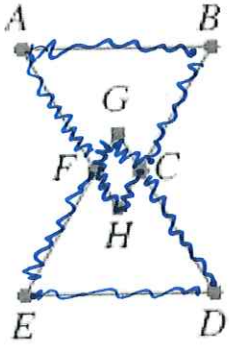
(A) Not a path
(B) A path
(C) A circuit
(D) An Euler path
(E) An Euler circuit

not adjacent

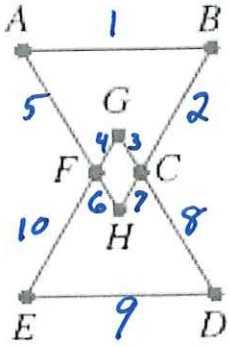


Classify the following: ABCGFHCDEFA.

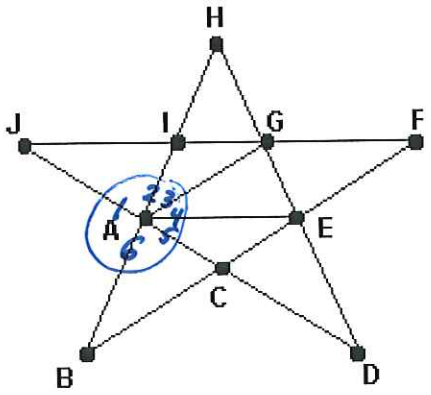
(A) Not a path
(B) A path
(C) A circuit
(D) An Euler path
(E) An Euler circuit



How many edges does the graph have? 10



What is the valence of vertex A in the graph below? 6



After a major natural disaster, such as a flood, hurricane, or tornado, many tasks need to be completed as efficiently as possible. For which situation below would finding an Euler circuit or an efficient eulerization of a graph be the appropriate mathematical technique to apply?

X (A) The electric company must check several substations for malfunctions.

(B) The gas company must check along all gas lines for possible leaks.

X (C) The phone company must respond to customers' needs in several parts of town.

X (D) The water company must spot-check the integrity of eight water towers located throughout the city.