CHAPTER 9 – VOTING

Plurality Method: Each voter votes for one candidate. The candidate with the most votes is the winner.

Majority Rule: Each voter votes for one candidate. The candidate with the majority of the votes wins.

When there are n votes, the majority is $\frac{n}{2}+1$ [n even] or $\frac{n+1}{2}$ [n odd]

Borda Count: Each voter ranks the n candidates with n points assigned to the first choice, $n-1$ to the second choice and so on. The candidate with the most points wins.

Hare Method: If there is no majority winner, then the candidate with the fewest number of first place votes is eliminated. The election is held again and if no majority winner is found, the candidate with the fewest first place votes is eliminated and the election held again. Repeat until a majority winner is found.

Pairwise Comparison Method: Each voter ranks the candidates. Each candidate is compared to each of the other candidates and the candidate who is preferred gets one point. The candidate with the most points wins.

Tournament Method: Compare the entire slate of candidates two at a time, in a pre-determined order. The candidate with the fewest votes is eliminated and the winner goes on to compare with the third candidate. These pairwise comparisons continue until a winner is found.

Approval Method: Each voter votes for all the candidates they approve of. The candidate with the most votes wins.

FAIRNESS CRITERIA:

- **Majority**: If a candidate receives a majority of the first place votes, then that candidate should be declared the winner.

- **Condorcet**: If a candidate is favored when compared one-on-one with every other candidate, then that candidate should be declared the winner.

- **Monotonicity**: A candidate who wins a first election and then gains additional support without losing any of the original support should also win a second election.

- **Irrelevant Alternatives**: If a candidate is declared the winner of an election and in a second election one or more of the candidates is removed, then the previous winner should still be declared the winner.

Note that there may be a tie. With two candidates and an even number of votes, it is possible that each received $n/2$ votes. The method to break the tie should be in place before the election!

Ways to break a tie:
Flip a coin
Use the number of first place votes.
Introduce a new voter [the Senate uses the VP].

A set of 9 voters had the following preference list for 4 candidates:

Choices	ADCB	ABCD	BCDA	BCAD	CBDA	CDBA	DCBA
# votes	3	1	1	1	1	1	1

9 votes So we need more than $\frac{9}{2} = 4.5$ votes for majority

(a) Who is the majority winner? If there is no majority, find the plurality winner.

$A = 3+1 = 4$ $B = 1+1 = 2$ $C = 1+1 = 2$ $D = 1 = 1$

No majority winner. Plurality winner = A

(b) Find the Borda count winner.

$\boxed{1^{st}} \cdot 4$ $2^{nd} \cdot 3$ $3^{rd} \cdot 2$ $4^{th} \cdot 1$ = Total

A $\underbrace{(3+1)}_{4}\cdot 4 + 0 \cdot 3 + (1)\cdot 2 + \underbrace{(1+1+1+1)}_{4}\cdot 1 = 22$

B $\underbrace{(1+1)}_{2}\cdot 4 + \underbrace{(1+1)}_{2}\cdot 3 + \underbrace{(1+1)}_{2}\cdot 2 + (3)\cdot 1 = 21$

C $\underbrace{(1+1)}_{2}\cdot 4 + \underbrace{(1+1+1)}_{3}\cdot 3 + \underbrace{(3+1)}_{4}\cdot 2 + 0\cdot 1 = 25$ ☆ C wins

D $\underbrace{(1)}_{1}\cdot 4 + \underbrace{(3+1)}_{4}\cdot 3 + \underbrace{(1+1)}_{2}\cdot 2 + \underbrace{(1+1)}_{2}\cdot 1 = 22$

(c) Find the winner using the Hare method.

	Elim D + redistribute		Elim B + redistribute	
A = 4		= 4		= 4
~~B~~ = 2		= 2		
C = 2	+ 1	= 3	+ 2	= 5 ☆ C wins
~~D = 1~~				

From 1st place votes

still no majority

(d) Find the winner using the tournament method in the order ABCD

↖ sequential pairwise

A vs B: 3+1 = 4 vs 1+1+1+1+1 = 5. B wins to face C

B vs C: 1+1+1 = 3 vs 3+1+1+1 = 6. C wins to face D

C vs D: 1+1+1+1+1 = 5 vs 3+1 = 4. C wins

(e) Comment on your results

winner may change depending on voting method

Seventeen board members vote on four candidates, A, B, C, or D, for a new position on their board. Their preference schedules are shown below.

Choices	~~A~~BCD	D~~A~~BC	~~C~~BD~~A~~
# votes	7	6	4

= 17 voters

we need more than $\frac{17}{2} = 8.5$ votes to win majority

we need 9 or more votes to win majority

(a) Find the winner using the Hare method.

A = 7 Elim B then elim C = 7

~~B~~ = 0

~~C~~ = 4

D = 6 + 4 = 10 D wins

(b) Using the Hare method according to the preference schedules shown, what happens if A rejects the offer before the ranking?

B = 7 + 4 Elim C + redistribute = 11 B wins

~~C~~ = 4

D = 6 = 6

(c) Comment on your results.

Hare method can violate the irrelevant alternatives Fairness Criteria

Consider the following preference table:

Choices	ABC	BCA	CBA
# votes	3	2	2

= 7 votes

A = 0 pts

B = 2 pts

C = 1 pt B wins

Pairwise comparison

Which candidate, if any, is the Condorcet winner?

A vs B: 3 vs 2+2; 3 vs 4

A vs C: 3 vs 2+2; 3 vs 4

B vs C: 3+2 vs 2; 5 vs 2

An election has 13 voters and their preferences are given in the table:

Choices	ABC	CBA	BCA	~~BAC~~ ABC
# votes	5	4	3	1

13 voters

We need more than $\frac{13}{2} = 6\frac{1}{2}$ votes for majority.
We need 7 or more votes for majority

(a) Determine the winner using the Hare method.

$A = 5 \quad = 5$

$B = 3+1 \quad = 4$ tie

$C = 4 \quad = 4$

how to break tie?

(b) Determine the winner using the Hare method if the last voter changes their preference from BAC to ABC.

Elim B & redistribute

$A = 5 + 1 \quad = 6 \qquad = 6$

$\cancel{B} = 3 \quad = 3$

$C = 4 \quad = 4 \quad + 3 \qquad = 7$

C wins

(c) Comment on your results.

Tie breakers are important
and
Hare method can violate monotonicty fairness criterion.

SAMPLE EXAM QUESTIONS FROM CHAPTER 9

1. Use the chart below to determine what kind of poker will be played if each player marks all the games he approves of and the approval method is used to determine the winner.

Game played is 7 card stud

people 8 voters

	A	B	C	D	E	F	G	H	
5 card draw	x		x	x					3
5 card stud	x			x		x		x	4
7 card stud		x		x	x	x	x		5
Texas hold 'em	x			x	x		x		4
Omaha hold 'em			x						1

2. Use the chart below to determine who which two people (top 2) will be elected to represent a class of 45 students to meet with the professor.

45 voters

	12	4	9	6	1	7	6	
Jeanetta	x			x			x	12+6+6 = 24
Mittie		x	x	x			x	4+9+6+6 = 25 *
Wilton	x		x		x		x	12+9+1+6 = 28 *
Jamaal	x	x						12+4 = 16
Yong		x	x			x		4+9+7 = 20

People chosen are Wilton and Mittie

3. Suppose that a nine-member committee needs to elect one of the four alternatives A, B, C, or D. Their preference schedule is shown below.

Choices	ABCD	BDAC	CDAB
# votes	4	3	2

9 votes

(a) Who wins using the Borda count?

(A) A (B) B (C) C (D) D (E) No winner

b/c don't have a tie breaker

$1^{st} \cdot 4 \quad 2^{nd} \cdot 3 \quad 3^{rd} \cdot 2 \quad 4^{th} \cdot 1 \quad = \text{Total}$

A $(4)\cdot 4 + (3+2)\,2 = 26$

B $(3)\cdot 4 + (4)\cdot 3 + (2)\,1 = 26$

C $(2)\cdot 4 + (4)\,2 + (3)\,1 = 19$

D $+ (3+2)\cdot 3 + (4)\,1 = 19$

(b) Who wins if a tournament is held with the ordering ABCD?
sequential pairwise

(A) A (B) B (C) C (D) D (E) No winner

A vs B	A vs C	A vs D
4+2 vs 3	4+3 vs 2	4 vs 3+2
6 vs 3	7 vs 2	4 vs 5

D wins

(c) Who wins using the Hare method?

(A) A (B) B (C) C (D) D (E) No winner

A = 4 + 2 = 6 ← majority, so A wins

B = 3 = 3

~~C~~ = 2 b/c D already out

~~D~~ = 0

Elim D, then Elim C + redistribute

4. There are 4 voters for 3 candidates. Their preference schedule is shown below.

BCA part d

4 votes, so we need more than $\frac{4}{2}=2$

majority is 3

Choices	ABC	BCA	CBA
# votes	2	1	1

(a) Determine the winner using the Hare method.

$A = 2$

$\left.\begin{array}{l} B = 1 \\ C = 1 \end{array}\right\}$ tie

(b) Determine the winner using the Borda count.

1st $\cdot 3$ + 2nd $\cdot 2$ + 3rd $\cdot 1$

$A \quad 2\cdot 3 + (1+1)\cdot 1 = 8$

$B \quad 1\cdot 3 + (2+1)\cdot 2 = 9$ ☆ B wins

$C \quad 1\cdot 3 + 1\cdot 2 + (2)\cdot 1 = 7$

(c) Determine the pairwise winner.

A vs B
2 1+1
2 2
Tie
½ pt each

A vs C
2 1+1
2 2
Tie
½ pt each

B vs C
2+1 1
3 1

$A = \frac{1}{2}+\frac{1}{2} = 1$

$B = \frac{1}{2}+1 = 1\frac{1}{2}$

$C = \frac{1}{2} = \frac{1}{2}$

B wins

(d) Determine the winner with the Hare method if the voter who liked the order CBA changes to BCA.

$A = 2 \quad = 2$ A & B tie

$B = 1+1 \quad = 2$

$C = \quad = 0$

5. The results of an election are

(ABCD)	(ABDC)	(CDAB)	(CDBA)	(DACB)
10	9	8	7	6

40 votes so need more than $\frac{40}{2} = 20$ votes for majority, so need 21 or more votes for majority

(a) Who is the majority winner? If there is no majority winner, use the Hare method to determine the winner.

No majority winner.

(A) A (B) B (C) C (D) D (E) No winner

Elim D & redistribute

$A = 10+9 = 19 + 6 = 25$ A wins using Hare method

$B =$

$C = 8+7 = 15 = 15$

$D = 6 = 6$

(b) Who is the Borda count winner?

(A) A (B) B (C) C (D) D (E) No winner

$1^{st} \cdot 4 + 2^{nd} \cdot 3 + 3^{rd} \cdot 2 + 4^{th} \cdot 1 = \text{Total}$

A $(10+9)\cdot 4 + (6)\cdot 3 + (8)\cdot 2 + (7)\cdot 1 = 117$ A wins

B $+ (10+9)\cdot 3 + (7)\cdot 2 + (8+6)\cdot 1 = 85$

C $(8+7)\cdot 4 + (10+6)\cdot 2 + (9)\cdot 1 = 101$

D $(6)\cdot 4 + (8+7)\cdot 3 + (9)\cdot 2 + (10)\cdot 1 = 97$

(c) Who is the pairwise comparison winner?

(A) A (B) B (C) C (D) D (E) No winner

A vs B: $10+9+8+6$ = 33; 7 = 7

A vs C: $10+9+6$ = 25; $8+7$ = 15

A vs D: $10+9$ = 19; $8+7+6$ = 21

B vs C: $10+9$ = 19; $8+7+6$ = 21

B vs D: $10+9$ = 19; $8+7+6$ = 21

C vs D: $10+8+7$ = 25; $9+6$ = 15

A = 2
B = 0
C = 2
D = 2

No winner
Tie

(d) Determine the winner in the following three tournaments. Note that ABCD means that A plays B and the winner advances to play C. The winner of that second contest plays D to determine the winner of the tournament.

(i) ABCD. Winner is D

A v B = A
A v C = A
A v D = D

(ii) DABC. Winner is C

D v A = D
D v B = D
D v C = C

(iii) BDCA. Winner is A

B v D = D
D v C = C
C v A = A

(e) Comment on your results.

Different tournament orders can give different winners