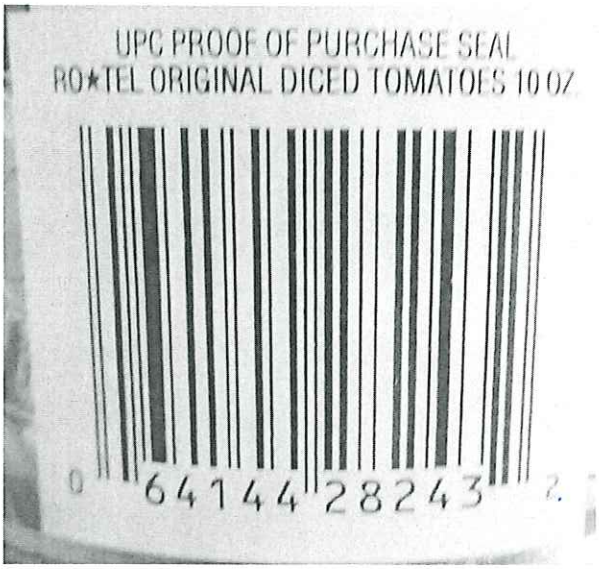
CHAPTER 16 – IDENTIFICATION NUMBERS

Consider the UPC code on a can of RO★TEL tomatoes



The scanner is not working so the clerk enters the numbers by hand as

0 64144 28263 2

and this is invalid even though the product code for the mild version of this is 28263. What happened?

The UPC codes use a ***check digit*** to minimize scanning errors. A check digit is a digit included in a code to help detect errors.

For a UPC code $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}a_{12}$ has a_{12} chosen so that the sum

$$3(a_1+a_3+a_5+a_7+a_9+a_{11})+1(a_2+a_4+a_6+a_8+a_{10}) \quad +a_{12}$$

is evenly divisibly by 10. What is the check digit for the mild RO★TEL?

a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10} a_{11}

0 6 4 1 4 4 2 8 2 6 3 _

$3(0+4+4+2+2+3)+1(6+1+4+8+6)=$

$45 + 25 = 70$

choose a_{12} so $70 + a_{12}$ is evenly divisible by 10

So $a_{12} = 0$

The numbers 20, 60, and 100 are all evenly divisible by 10 so can we find a way to talk about numbers when we only care about the remainders?

Yes . Modulo

Definition: Congruence Modulo m

Let a, b, and m be integers with $m \geq 2$. Then a is congruent to b modulo m, written

$$a \equiv b \bmod m$$

means that m evenly divides $a-b$.

Determine if the congruences below are true or false:

$25 \equiv 1 \bmod 6$ $\frac{25-1}{6}$ is remainder = 0? True

$100 \equiv 20 \bmod 10$ $\frac{100-20}{10}$ Is remainder 0? True

$52 \equiv 0 \bmod 13$ $\frac{52-0}{13}$ Is remainder 0? True

$75 \equiv 7 \bmod 5$ $\frac{75-7}{5}$ Is remainder 0? False

$5\overline{)68}$ 13, -5, 18, -15, 3 ← remainder of 3

x* mod *y is equal to the remainder when you divide x by y.

Find the following values:

(a) 34 mod 5 = 4 ← $5\overline{)34}$ 6, -30, 4 ← remainder

(b) 78 mod 11 = 1 ← $11\overline{)78}$ 7, -77, 1 ←

(c) 13 mod 15 = 13 ← $15\overline{)13}$ 0, -0, 13

(d) 12 mod 2 = 0 ← $2\overline{)12}$ 6, -12, 0 ←

Types of errors when dealing with identification numbers:

- Replacing one digit with a different digit (single digit error)
  - *ac* entered rather than *ab* Intended 94 entered 96
- Transposing two adjacent digits (adjacent transposition error)
  - *ba* entered rather than *ab* Intended 94 entered 49
- Transposing a sequence of digits (jump transposition error)
  - *cba* entered rather than *abc* Intended 147 entered 741

Note that some of the digits in the UPC code are multiplied by 3. Those digits had a ***weight*** of 3.

A code $a_1a_2a_3a_4a_5a_6$ (check digit) uses the last digit as a check digit. The check digit is found using the formula

$$a_6 = a_1 + a_3 + 3(a_2 + a_4) \bmod 10$$

(a) What is the check digit for the code 23714?

$a_1 a_2 a_3 a_4 a_5$

$a_6 = 2 + 7 + 3(3 + 1)$
$= 2 + 7 + 3(4)$
$= 2 + 7 + 12$
$= 21$

21 mod 10 = 1

$10\overline{)21}$ 2, -20, 1

$a_1 a_2 a_3 a_4 a_5 a_6$

(b) Find the value of the missing digit x in the code 46x782

$2 = (4 + x + 3(6 + 7)) \bmod 10$
$4 + x + 3(13)$
$4 + x + 39$
$x + 43$

$x + 43$ should give remainder of 2 when divided by 10

$52 = x + 43$
$-43 \quad -43$
$9 = x$

$x = 9$

(c) Will this code find an error if the first digit is entered incorrectly?

p 3
c)

$a_6 = a_1 + a_3 + 3(a_2 + a_4) \bmod 10$

$e_6 = e_1 + a_3 + 3(a_2 + a_4) \bmod 10$

Will <u>not</u> detect error if $a_6 - e_6$ is a multiple of 10

$(a_1 + a_3 + 3(a_2 + a_4)) - (e_1 + a_3 + 3(a_2 + a_4)) =$ a multiple of 10

$a_1 + a_3 + 3(a_2 + a_4) - e_1 - a_3 - 3(a_2 + a_4) =$ a multiple of 10

$a_1 - e_1 =$ a multiple of 10

$|a_1 - e_1| =$ a multiple of 10

$|a_1 - e_1|$: digits

a multiple of 10: 0, 10, 20, 30, 40, 50, ...

if $|a_1 - e_1| = 0$

$a_1 - e_1 = 0$

$a_1 = e_1$

I entered the correct # so no error

if $|a_1 - e_1| = 10$

Two digits can not be further apart than 9

No error

This catches all single-digit errors in 1st digit

A code is given by $a_1a_2a_3$ where a_3 is the check digit and (a_3: check digit)

$$a_3 = a_1 + 4\,a_2 \mod 9$$

we intended $a_1\ a_2$
we entered $a_2\ a_1$

$a_3 = (a_1 + 4a_2)$ $\quad e_3 = a_2 + 4a_1$

(a) Will this check digit find all transposition errors?

will not catch error if

$|(a_1+4a_2) - (a_2+4a_1)|$ = multiple of 9

$|a_1 + 4a_2 - a_2 - 4a_1|$ = multiple of 9

$|3a_2 - 3a_1|$ = multiple of 9

$3\,|a_2 - a_1|$ = multiple of 9

$|a_2 - a_1|$ = multiple of $\frac{9}{3}$

$|a_2 - a_1|$ = mul of 3

when $|a_2 - a_1| = 0$

$a_2 - a_1 = 0$

$a_2 = a_1$ ← both digits the same, so no error

when $|a_2 - a_1| = 3$, $|a_2 - a_1| = 6$, $|a_2 - a_1| = 9$ } will not catch

(b) Will this check digit find all single digit errors in the first position?

$a_3 = (a_1 + 4a_2) \mod 9$ $\quad e_3 = (e_1 + 4a_2) \mod 9$

not catch errors if $|(a_1+4a_2) - (e_1+4a_2)|$ = multiple of 9

$|a_1 + 4a_2 - e_1 - 4a_2|$ = multiple of 9

$|a_1 - e_1|$ = multiple of 9

if $|a_1 - e_1| = 0$
No error, entered correct #

if $|a_1 - e_1| = 9$
not catch error

(c) Will this check digit find all single digit errors in the second position?

$a_3 = (a_1 + 4a_2) \mod 9$ $\quad e_3 = (a_1 + 4e_2) \mod 9$

not catch errors if $|(a_1+4a_2) - (a_1+4e_2)|$ = multiple of 9

$|a_1 + 4a_2 - a_1 - 4e_2|$ = multiple of 9

$|4a_2 - 4e_2|$ = multiple of 9

$4\,|a_2 - e_2|$ = multiple of 9

$|a_2 - e_2|$ = multiple of $\frac{9}{4}$

if $|a_2 - e_2| = 0$
$a_2 - e_2 = 0$
$a_2 = e_2$
no error, we entered correct #.

if $|a_2 - e_2| = \frac{9}{4}$
Impossible b/c digits are separated by integers

if $|a_2 - e_2| = \frac{18}{4}$ Impossible

if $|a_2 - e_2| = \frac{27}{4}$ Impossible

if $|a_2 - e_2| = \frac{36}{4} = 9$ ← not catch error

Data can be encoded in identification numbers.

March 17 → 2 full months + 17 days → 31(2) + 17

The last 5 digits of Illinois driver's license numbers are based on the driver's birthday and gender. For a man, the last 5 digits are the birth year followed by the day of the year based on each month having 31 days. For a woman, 600 is added to the number.

Man = $31(m-1)+d$

Woman = $31(m-1)+d+600$

_ _ | _ _ _

Birth Year

(a) What would the last 5 digits of an of Illinois driver's license number look like for a man born on February 12, 1967?

$m=2$ $d=12$

6 7 0 4 3

Man = $31(2-1)+12$

$= 31(1)+12$

$= 31+12$

$= 43$

(b) What do you know about a person who has the last 5 digits 10642?

Woman = $31(m-1)+d+600 = 642$

$-600 \quad -600$

$31(m-1)+d = 42$

b/c 42 is between 32 & 62

$31(2-1)+d = 42$

$31 + d = 42$

$-31 \quad -31$

$d = 11$

1910 or 2010 ← Female

Feb 11 1910 or 2010

too young to drive

(c) What do you know about a person who has the last 5 digits 90373?

1990 male

Male = $31(m-1)+d = 373$ ← big #, so try December

$31(12-1)+d = 373$

$31(11)+d = 373$

$341 + d = 373$

$-341 \quad -341$

$d = 32$

Dec 32, 1990 ????

Error ....

Fake ID??

SAMPLE EXAM QUESTIONS FROM CHAPTER 16

1. Determine the check digit that should be appended to the identification number 634498, if the check digit is the number needed to bring the total of all the digits to a multiple of 10.

(A) The code is invalid (B) 6 (C) 8 (D) 4

(E) None of these

$6+3+4+4+9+8+d$ ← check digit $=$ multiple of 10

$34+d =$ multiple of 10

$34+d = 40$

2. Which, if any, of the statements below are true? Mark all correct answers.

T (A) $101 \equiv 1 \bmod 2$ $\frac{101-1}{2}$ remainder 0?

T (B) $77 \equiv 0 \bmod 11$ $\frac{77-0}{11}$ remainder 0?

$11\overline{)77}$ 7, -77, 0

T (C) $49 \equiv 1 \bmod 12$ $\frac{49-1}{12}$ remainder 0?

$12\overline{)48}$ 4, -48, 0

F (D) $39 \equiv 5 \bmod 5$ $\frac{39-5}{5}$ remainder 0!

$5\overline{)34}$ 6, -30, 4

(E) None of these are true.

3. The number 4320 is accidentally entered as 4321.

What type of error is this?

(A) A transposition error

(B) A jump transposition error

(C) A single digit error

(D) A baseball error

(E) None of these

4. The last three digits of a man's ID number are the birth day of the year based on each month of the year having 35 days. If the person is a woman, 500 is added to the birth day.

Male $= 35(m-1)+d$

Female $= 35(m-1)+d+500$

(a) What are the last three digits of a man's ID number if he was born on October 8th?

$m=10 \quad d=8$

$$\text{Male} = 35(10-1)+8$$
$$= 35(9)+8$$
$$= 315+8$$
$$= 323$$

(b) What do you know about a person if the last three digits of the person's ID number are 503?

Female January 3

$$\text{Female} = 35(m-1)+d+500 = 503$$
$$-500 \qquad -500$$
$$35(m-1)+d = 3$$

too small for any complete months to have passed

(c) What do you know about a person if the last three digits of the person's ID number is 420? Male

$$35(m-1)+d = 420$$

Try December

$$35(12-1)+d = 420$$

$$35(11)+d = 420$$
$$385+d = 420$$
$$-385 \qquad -385$$
$$d = 35$$

Dec 35 ??? Fake

5. A code is given by $a_1a_2a_3a_4$ where a_4 is the check digit. The check digit is $a_4 = 7a_1 + 2a_2 + 5a_3 \bmod 9$.

(a) Determine the value of x in the code $2x45$, given that the check digit is valid.

$a_1\ a_2\ a_3\ a_4$ $\quad x = 8$

$5 = (7(2) + 2(x) + 5(4)) \bmod 9$

$5 = (14 + 2x + 20) \bmod 9$

$5 = (34 + 2x) \bmod 9$

$\frac{34 + 2x - 5}{9}$ remainder of 0

$\frac{29 + 2x}{9}$ remainder of 0

$29 + 2x$ is multiple of 9

$29 + 2(8) = 45$

Try $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

0, 9, 18, 27, 36, 45, 54, 63, 72, 81, ...

(b) Determine if the check digit will find all single digit errors in the third position.

$a_4 = (7a_1 + 2a_2 + 5a_3) \bmod 9$ $\quad e_4 = (7a_1 + 2a_2 + 5e_3) \bmod 9$

Not catch error

$|(7a_1 + 2a_2 + 5a_3) - (7a_1 + 2a_2 + 5a_3)| =$ multiple of 9

$|7a_1 + 2a_2 + 5a_3 - 7a_1 - 2a_2 - 5a_3| =$ multiple of 9

$|5a_3 - 5e_3| =$ multiple of 9

$5\,|a_3 - e_3| =$ multiple of 9

$|a_3 - e_3| =$ multiple of $\frac{9}{5}$

$|a_3 - e_3| = 0$? No error

$|a_3 - e_3| = \frac{9}{5}$ Impossible

$|a_3 - e_3| = \frac{18}{5}$ Impossible

$|a_3 - e_3| = \frac{27}{5}$ Impossible

$|a_3 - e_3| = \frac{36}{5}$ Impossible

$|a_3 - e_3| = \frac{45}{5} = 9$ ← not catch

(c) Determine if the check digit will find all transposition errors in the second and third positions.

$a_4 = (7a_1 + 2a_2 + 5a_3) \bmod 9$ $\quad e_4 = (7a_1 + 2a_3 + 5a_2) \bmod 9$

Not catch error

$|(7a_1 + 2a_2 + 5a_3) - (7a_1 + 2a_3 + 5a_2)| =$ multiple of 9

$|7a_1 + 2a_2 + 5a_3 - 7a_1 - 2a_3 - 5a_2| =$ mul of 9

$|3a_3 - 3a_2| =$ mul of 9

$3\,|a_3 - a_2| =$ mul of 9

$|a_3 - a_2| =$ mul of $\frac{9}{3}$

$|a_3 - a_2| =$ mul of 3

$|a_3 - a_2| = 0$ No error b/c same #

$|a_3 - a_2| = 3$

$|a_3 - a_2| = 6$

$|a_3 - a_2| = 9$

not catch