

# Pythagoras Proofs

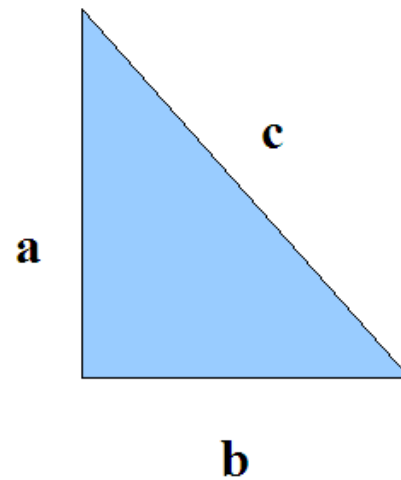
TEKS 8.07 (C):

The student is expected to use pictures or models to demonstrate the Pythagorean Theorem

## **The Pythagorean Theorem:**

In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.

### **“Traditional Textbook Proof” (Legendre/Bhaskara)**



## The Pythagorean Theorem:

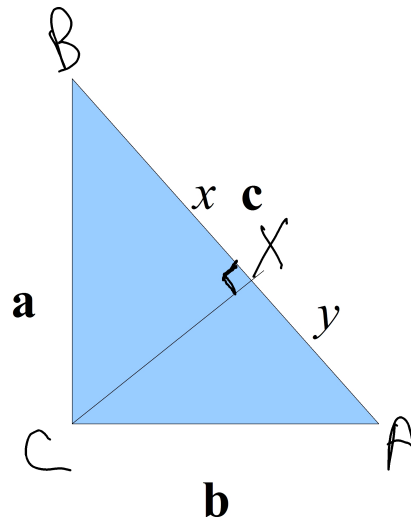
In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.

### “Traditional Textbook Proof” (Legendre/Bhaskara)

similar  $\triangle$  BXC and BCA

$$\frac{a}{x} = \frac{c}{a}$$

$$a^2 = cx$$



similar  $\triangle$  AXA and ACB

$$\frac{b}{y} = \frac{c}{b}$$

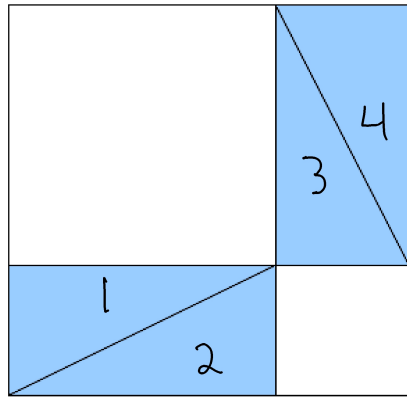
$$b^2 = cy$$

$$\begin{aligned} \therefore a^2 + b^2 &= cx + cy \\ &= c(x+y) \\ a^2 + b^2 &= c^2 \quad \checkmark \end{aligned}$$

# Puzzle Proof #1

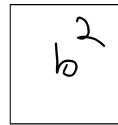
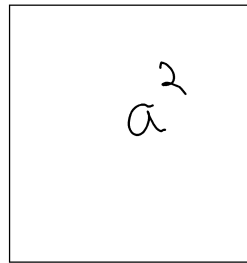
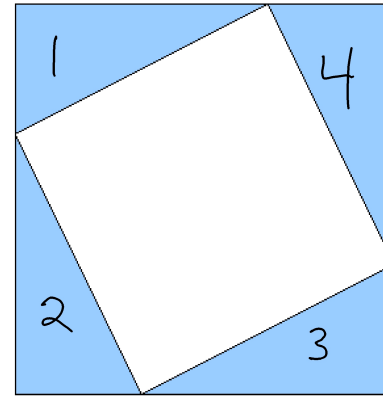
- Using the small square, the medium square, and four triangles, make another square.
- Repeat step one using the large square and four triangles.
- Compare the sizes of the two squares-what do you observe?

In the space below, draw your two squares, one on each side of the equals sign. Place corresponding numbers in the pieces which are the same in both squares. Draw the “equation” that remains when you remove these pieces from both sides.



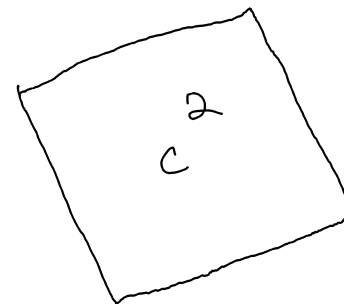
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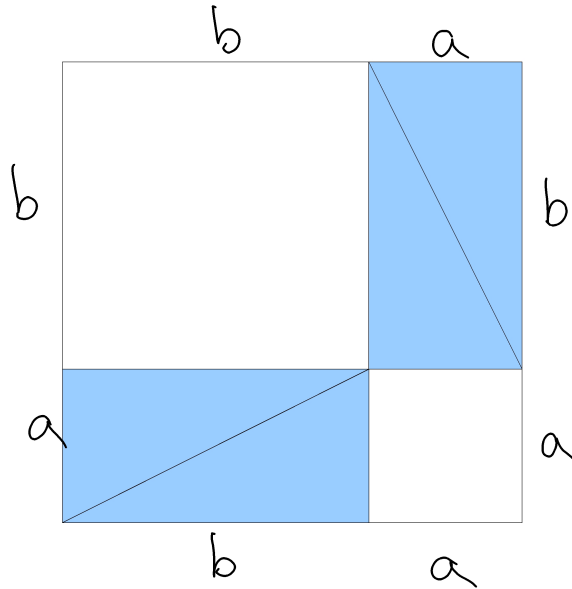


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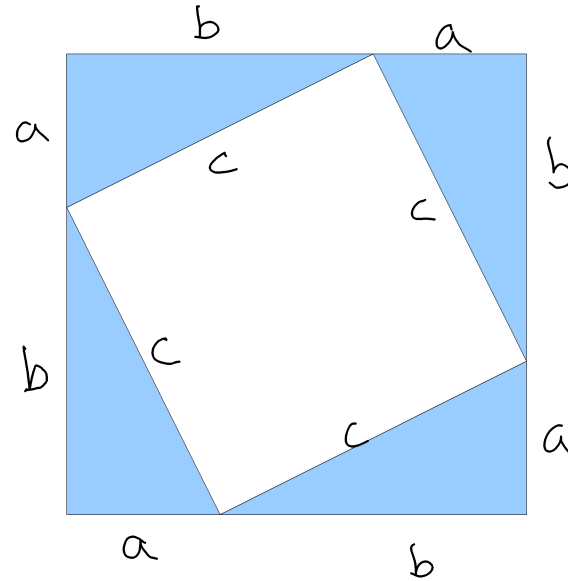


# Formal Proof:



$$\begin{aligned} \text{Area} &= (a+b)^2 \quad (\text{Square}) \\ \text{Area} &= a^2 + b^2 + 4\left(\frac{1}{2}ab\right) \quad (\text{Pieces}) \\ &= a^2 + b^2 + \cancel{2ab} \end{aligned}$$

$$a^2 + b^2$$

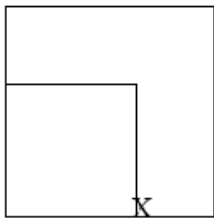


$$\begin{aligned} \text{Area} &= (a+b)^2 \quad (\text{Congruent Square}) \\ \text{Area} &= c^2 + 4\left(\frac{1}{2}ab\right) \quad (\text{Pieces}) \\ &= c^2 + \cancel{2ab} \end{aligned}$$

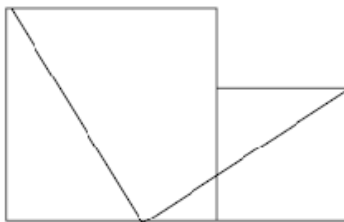
$$= c^2 \quad \checkmark$$

# Puzzle Proof #2

**I.** Use the smallest two squares for the following activity. Place the smaller square so that it fits in the lower left corner of the larger square. Carefully mark the spot on the larger square where the lower right corner of the smaller square is. (see figure below)



**II.** Now tape the two squares together as shown below and draw lines from the top left and top right corners to the mark you made in the previous step



**III.** Cut along these two lines. You should have three pieces now. Try to form a single square using these pieces. Draw your figure below:

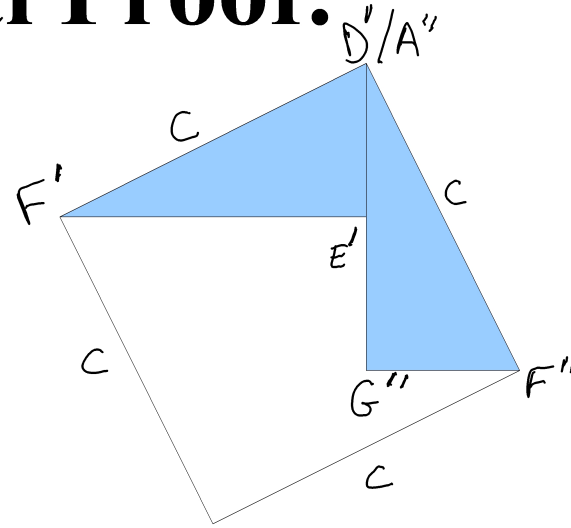
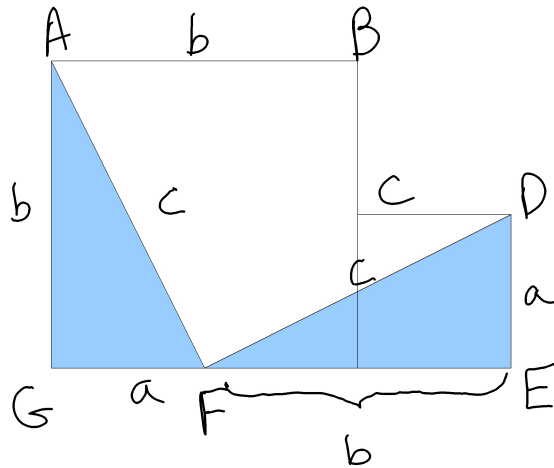
If the original squares had side lengths  $a$  and  $b$ , what is the total area of the figure in part II?

What is the area of the square in part III?

Are these areas equal? Why or why not?



# Formal Proof:



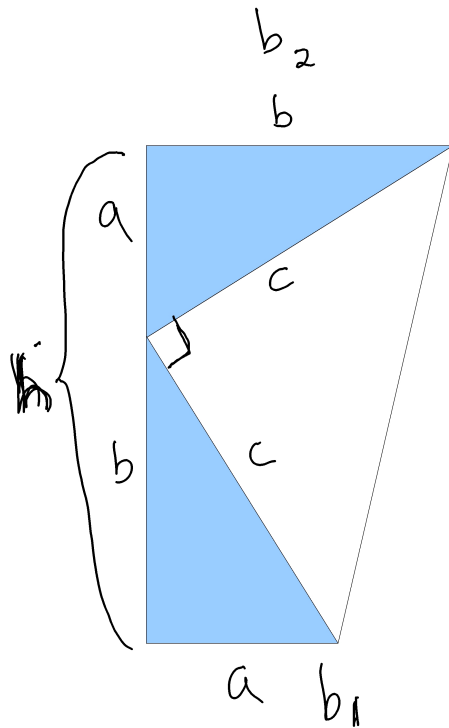
Right Angle?

$$\begin{aligned} \angle AFG + \angle GAF &= 90 \\ \angle GAF &= \angle FED \text{ (congruent } \triangle) \\ \angle AFG + \angle FED &= 90 \text{ but} \\ \angle AFG + \angle AFD + \angle FED &= 180 \\ \text{so } \angle AFD &= 90^\circ \end{aligned}$$

OR

$$\begin{aligned} \angle F'D'E' + \angle D'E'F'' &= 90 \\ \angle D'E'F'' &= \angle G''A''F'' \text{ (congruent } \triangle) \\ \text{so } \angle F'D'E' + \angle G''A''F'' &= 90^\circ \end{aligned}$$

# Proof #3



Trapezoid:

$$\begin{aligned}
 A &= \frac{1}{2}(b_1+b_2)h \\
 &= \frac{1}{2}(a+b)(a+b) \\
 &= \frac{1}{2}a^2 + \cancel{ab} + \frac{1}{2}b^2
 \end{aligned}$$

Pieces:

$$\begin{aligned}
 A &= \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2 \\
 &= \frac{1}{2}c^2 + \cancel{ab}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{2}a^2 + \frac{1}{2}b^2 &= \frac{1}{2}c^2 \\
 a^2 + b^2 &= c^2 \checkmark
 \end{aligned}$$

**For more proofs, visit:**

**<http://www.cut-the-knot.org/pythagoras/>**