Name________________________UIN__________________

MATH 171 Exam 1 Fall 2021

Sections 503 P. Yasskin

1-10	/50	12	/25
11	/ 5	13	/25
		Total	/105

Multiple Choice: (5 points each. No part credit.)

1. Write $\langle 1,5\rangle$ as a linear combination of $\langle 2,3\rangle$ and $\langle 3,1\rangle$ or type "impossible" in both boxes.

$$\langle 1,5\rangle = \_\_\_\_\_\_\langle 2,3\rangle + \_\_\_\_\_\_\langle 3,1\rangle$$

2. Find the angle between the vectors $\langle 2,3\rangle$ and $\langle 5,1\rangle$.

a. $0°$

b. $30°$

c. $45°$

d. $60°$

e. $90°$

f. $120°$

g. $135°$

h. $150°$

i. $180°$

3. Write $\vec{v} = \langle 10,5\rangle$ as the sum of two vectors $\vec{p}$ and $\vec{q}$ where $\vec{p}$ is parallel to $\vec{u} = \langle 3,4\rangle$ and $\vec{q}$ is perpendicular to $\vec{u}$.

$$\langle 10,5\rangle = \vec{p} + \vec{q}$$

where

$$\vec{p} = \langle \_\_\_\_\_\_, \_\_\_\_\_\_ \rangle \quad \text{and} \quad \vec{q} = \langle \_\_\_\_\_\_, \_\_\_\_\_\_ \rangle$$

4. Find the smallest interval with integer endpoints in which there is a solution of the equation $x^3 + 3x = 40$.

There is a solution in the interval $I = [\_\_\_\_\_\_, \_\_\_\_\_\_]$.

5. For the piecewise defined function $f(x) = \begin{cases} 5 & \text{for } x = 4 \\ 9 - x & \text{for } x > 4 \\ 2 + x & \text{for } x < 4 \end{cases}$ identify

$f(4) =$ ______ $\qquad \lim_{x \to 4^-} f(x) =$ ______ $\qquad \lim_{x \to 4^+} f(x) =$ ______

Then enter T or F to say if each statement is true or false:

a. $f(x)$ is continuous ______ (TorF)

b. $f(x)$ is continuous from the right ______ (TorF)

c. $f(x)$ is continuous from the left ______ (TorF)

d. $\lim_{x \to 4} f(x)$ exists ______ (TorF)

6. Find the interval on which $g(x) = \dfrac{1}{\sqrt{4 - x^2}} + \sqrt{x}$ is continuous.

a. $-2 < x \leq 0$

b. $-2 \leq x \leq 2$

c. $2 < x < \infty$

d. $0 \leq x < 2$

e. $0 < x \leq 2$

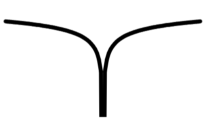
7. Find the horizontal asymptotes for $g(x) = \dfrac{5 \cdot 3^x + 4}{3^x + 2}$.

As $x \to +\infty$, the horizontal asymptote is $y =$ ______

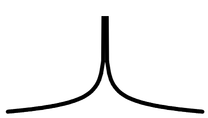
As $x \to -\infty$, the horizontal asymptote is $y =$ ______

8. The function $f(x) = \dfrac{x - 4}{(x - 2)^2}$ has a vertical asymptote at $x = 2$. Near $x = 2$, its graph looks like:

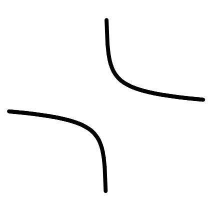
a.



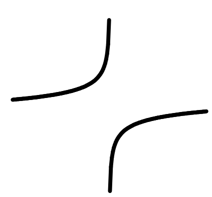
b.



c.



d.



e None of these

9. Find the average velocity between $t_1 = 1$ and $t_2 = 1.1$ if the position is $x(t) = t^2$.

a. 12.01

b. 12.1

c. 2

d. 2.01

e. 2.1

10. Find the tangent line to the curve $y = \dfrac{1}{x^2}$ at $x = 2$. It can be written in slope intercept form as $y = mx + b$, where

$m =$ ______ and $b =$ ______

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (5 points) Write out the definition of the statement $\lim_{x\to 4} x^3 = 64$.
Your answer must consist of words, phrases and formulas from the following list:

For	and	$\varepsilon > 0$	$\delta > 0$
such that	or	$\lvert x - 4\rvert < \varepsilon$	$\lvert x - 4\rvert < \delta$
there exists	if	$0 < \lvert x - 4\rvert < \varepsilon$	$0 < \lvert x - 4\rvert < \delta$
there does not exist	then	$\lvert x^3 - 64\rvert < \varepsilon$	$\lvert x^3 - 64\rvert < \delta$
some	all	$0 < \lvert x^3 - 64\rvert < \varepsilon$	$0 < \lvert x^3 - 64\rvert < \delta$

$\lim_{x\to 4} x^3 = 64$ means:

12. (25 points) Compute each of the following limits.

a. $\lim_{k \to 4} \frac{k-4}{k^2-k-12} =$

b. $\lim_{x \to 5} \frac{(x-10)^2-25}{x-5} =$

c. $\lim_{x \to 4} \frac{x-4}{\sqrt{12+x}-\sqrt{20-x}} =$

d. $\lim_{x \to \infty} \left(x - \frac{x^2+3}{x+4} \right) =$

e. $\lim_{\theta \to 0} \frac{1-\cos^4\theta}{\theta^2} =$

13. (25 points) Compute the derivative of each of the following functions.

a. $f(x) = 5x^4 - 3x^2 + 7x - \dfrac{2}{x^3}$

b. $g(y) = y^3 \cos(y)$

c. $h(t) = \dfrac{\sin(t)}{t}$

d. $k(x) = 2x^e + 3e^x$

e. If $f(x) = \dfrac{p(x) + q(x)}{r(x)}$, find $f'(1)$, given that

$$p(1) = 7, \quad p'(1) = 6, \quad q(1) = 5, \quad q'(1) = 4, \quad r(1) = 3, \quad r'(1) = 2$$