Name________________________UIN__________________

1-10	/55	12	/10
11	/25	13	/15
		Total	/105

MATH 171 Exam 2 Fall 2021

Sections 503 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. If $f(x) = e^{3x} - 4e^{2x} + 2x^4 + \sin(2x) + \ln(1-3x)$, find $f'(0)$.

$f'(0) =$ ____________

2. If $F(x) = f'(g(x))$, where

$$\begin{array}{llll} f(2) = 2 & f'(2) = 3 & g(2) = 4 & g'(2) = 5 \\ f(4) = 4 & f'(4) = 5 & g(4) = 6 & g'(4) = 7 \end{array}$$

find $F'(2)$.

$F'(2) =$ ____________

3. If $g(x) = x\cos(\pi x)$, find $g'\left(\frac{1}{2}\right)$. (Type "pi" for π. Type "sqrt(3)" for $\sqrt{3}$.)

$g'\left(\frac{1}{2}\right) =$ ____________

4. Find the slope of the curve $xy^2 + x^2y^3 = 6$ at the point $(x,y) = (2,1)$.

$\left.\frac{dy}{dx}\right|_{(2,1)} =$ ____________

5. (10 points) Consider the parametric curve $\vec{r}(t) = \langle t^3 + 3t, t^3 - 3t \rangle$.

a. Find the position at time $t = 2$.

$\vec{r}(2) = \langle$__________,__________$\rangle$

b. Find the velocity at time $t = 2$.

$\vec{v}(2) = \langle$__________,__________$\rangle$

c. Find the parametric tangent line at $t = 2$.
(Write each component in the form.$a + bt$ *with no spaces.)*

$x(t) =$ ____________________ $y(t) =$ ____________________

d. Find the slopes at times $t = 2$ and $t = 1$.

$m(2) =$ ____________ $m(1) =$ ____________

e. Find the time(s) at which the curve is horizontal.
(Put times in separate blanks in ascending order.)

$t =$ ____________ and $t =$ ____________

6. If $f(x) = \sqrt{25 - x^2} + \arcsin\left(\frac{x}{5}\right)$, then $f'(3) =$

a. 1

b. $\frac{1}{2}$

c. $-\frac{1}{4}$

d. $-\frac{1}{2}$

e. $-\frac{3}{4}$

7. Notice that the derivative of $f(x) = x + x^3 + x^5$ is always positive. So it is always increasing and is 1-to-1. So it has an inverse $g(x)$. Find $g'(3)$.

HINT: $f(-1) = -3$ $f(0) = 0$ $f(1) = 3$ $f(2) = 42$ $f(3) = 273$

$g'(3) =$ ____________

8. The distance from Houston to Dallas is 240 miles. The highest speed limit for the entire trip is $75 \frac{\text{miles}}{\text{hour}}$. An Aggie makes the trip in 3 hours. Which theorem says that the Aggie was speeding at some point along the trip?

a. The Squeeze Theorem
b. The Mean Value Theorem
c. The Intermediate Value Theorem
d. Rolle's Theorem

9. The side of a cube is measured to be $s = 20\,\text{cm} \pm 0.05\,\text{cm}$. So the volume of the cube is $V = s^3 \pm \Delta V = 8000\,\text{cm}^3 \pm \Delta V$. Using the linear approximation, what is the error ΔV in this computation of the volume.

$\Delta V \approx$ ____________ cm^3

10. If the position function is $x(t) = \sin(t^2)$, find the jerk at $t = 1$.
Note: the jerk is $j(t) = \frac{d^3x}{dt^3}$.

$j(1) =$ ____________ $\sin 1 +$ ____________ $\cos 1$

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (25 points) Consider the function $f(x) = \frac{1}{5}x^5 - x^4 + 3$.

a. (3 pts) Find $f'(x) =$ ______________

b. (3 pts) Find $f''(x) =$ ______________

c. (2 pts) Find all critical points of f, i.e all values of x at which $f'(x) = 0$.

critical points at: $x =$_______________

d. (3 pts) Find the intervals where f is increasing and decreasing. (If none, say none.)

increasing on: _______________ decreasing on: _______________

e. (2 pts) Find all secondary critical points of f, i.e all values of x at which $f''(x) = 0$.

secondary critical points at: $x =$_______________

f. (3 pts) Find the intervals where f is concave up and concave down. (If none, say none.)

concave up on: _______________ concave down on: _______________

g. (4 pts) What does the Second Derivative Test say about each critical point?

h. (3 pts) Find the x location of all local minima and local maxima of f. (If none, say none.)

local minima at: $x =$_______________ local maxima at: $x =$_______________

i. (2 pts) Find the x location of all inflection points of f. (If none, say none.)

inflection points at: $x =$_______________

12. (10 points) A weather balloon is currently at $x_0 = 2490$ meters from the weather station and currently has velocity $v = 4\,\dfrac{\text{meters}}{\text{hour}}$. The balloon measures the temperature is currently $T_0 = 78^\circ$F and has derivative $\dfrac{dT}{dx} = 0.2\,\dfrac{^\circ\text{F}}{\text{meter}}$.

a. (4 pts) What is $\dfrac{dT}{dt}$, i.e. the current time rate of change of the temperature?

b. (3 pts) What will be the approximate position x_1 of the balloon after $\frac{1}{2}$ hour?

c. (3 pts) What will be the approximate temperature T_1 at the location of the balloon after $\frac{1}{2}$ hour?

13. (15 points) Compute the derivatives of the following functions.

a. $p(t) = \sin^3(\cos(t^2))$

b. $g(x) = \csc(\arcsin(x^2))$ (There cannot be any trig or arctrig functions in your answer.)

c. $f(x) = \dfrac{(x-2)^{12}(x+1)^{10}}{(x-3)^8}$ Find $f'(1)$.